Geometry Putnam Seminar

1. Let ABCD be a square, and let be a line passing through the midpoint of segment AB that intersects segment BC. Given that the distances from A and C to are 4 and 7, respectively, compute the area of ABCD.

- 2. Let C be a point on a semicircle of diameter AB and let D be the midpoint of arc AC. Let E be the projection of D onto the line BC and F the intersection of line AE with the semicircle. Prove that BF bisects the line segment DE.
- 3. Let ABCDEF be a regular hexagon of side length 1, and O be the center of the hexagon. In addition to the sides of the hexagon, line segments are drawn from O to each vertex, making a total of twelve unit line segments. Find the number of paths of length 2003 along these line segments that start at O and terminate at O.
- 4. Let ABC be an equilateral triangle inscribed in a circle. Point D is added on the circle between A and C. What is AD + CD BD?
- 5. Let ABTCD be a convex pentagon with area 22 such that AB = CD and the circumcircles of triangles TAB and TCD are internally tangent. Given that $\angle ATD = 90^{\circ}$, $\angle BTC = 120^{\circ}$, BT = 4, and CT = 5, compute the area of triangle TAD.
- 6. Let ABC be a triangle and let D and E be points on the sides AB and AC, respectively, such that DE is parallel to BC. Let P be any point interior to triangle ADE, and let F and G be the intersections of DE with the lines BP and CP, respectively. Let Q be the second intersection point of the circumcircles of triangles PDG and PFE. Prove that the points A, P, and Q are collinear.
- 7. Suppose that there are 4 distinct points in the plane, no three of which are collinear, and such that the four points do not all lie on a common circle. Show that one point lies inside the circle through the other three.
- 8. Prove that no matter how 3 points are placed in the closed unit square, some two of them are distance $\leq \sqrt{6} \sqrt{2}$ apart.
- 2024 B2 Two convex quadrilaterals are called *partners* if they have three vertices in common and they can be labeled ABCD and ABCE so that E is the reflection of D across the perpendicular bisector of the diagonal \overline{AC} . Is there an infinite sequence of convex quadrilaterals such that each quadrilateral is a partner of its successor and no two elements of the sequence are congruent?
- 2024 A5 Consider a circle Ω with radius 9 and center at the origin (0,0), and a disc Δ with radius 1 and center at (r,0), where $0 \le r \le 8$. Two points P and Q are chosen independently and uniformly at random on Ω . Which value(s) of r minimize the probability that the chord \overline{PQ} intersects Δ ?
- 2019 A2 In the triangle $\triangle ABC$, let G be the centroid, and let I be the center of the inscribed circle. Let α and β be the angles at the vertices A and B, respectively. Suppose that the segment IG is parallel to AB and that $\beta = 2 \tan^{-1}(1/3)$. Find α .
- 2017 B5 A line in the plane of a triangle T is called an equalizer if it divides T into two regions having equal area and equal perimeter. Find positive integers a > b > c, with a as small as possible, such that there exists a triangle with side lengths a, b, c that has exactly two equalizers.