1. Example
Show that for all $t \in \mathbb{R}$,
$$e^t \ \geq \ 1+t,$$
with equality iff $t = 0$.

2. Example
For $a, b, c > 0$,
$$(a+b+c)\Big(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\Big) \ \geq \ 9.$$

3. Determine all ordered pairs of real numbers (a, b) such that the line $y = ax + b$ intersects the curve $y = \ln(1 + x^2)$ in exactly one point.

4. Prove that not all zeros of the polynomial $P(x) = x^4 - \sqrt{7}x^3 + 4x^2 - \sqrt{22}x + 15$ are real.

5. Holder's Inequality:
If $a_1 + a_2 + \cdots + a_n = n$, prove that $a_1^4 + \cdots + a_n^4 \geq n$.

6. Let $a, b, c, d > 0$ with $abcd = 1$. Prove
$$\big(ab+bc+cd+da\big)^3 \ \geq \ (a+b+c+d)^4.$$

7. Cauchy-Schwartz:
Find all positive integers $n, k_1, \ldots, k_n$ such that $k_1 + \cdots + k_n = 5n - 4$ and $\frac{1}{k_1} + \cdots + \frac{1}{k_n} = 1$.

8. 2023 A2
Let n be an even positive integer. Let p be a monic, real polynomial of degree $2n$; that is to say, $p(x) = x^{2n} + a_{2n-1}x^{2n-1} + \cdots + a_1x + a_0$ for some real coefficients $a_0, \ldots, a_{2n-1}$. Suppose that $p(1/k) = k^2$ for all integers k such that $1 \leq |k| \leq n$. Find all other real numbers x for which $p(1/x) = x^2$.

9. 2009 A1
Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

10. 2008 B2
Let $F_0(x) = \ln x$. For $n \geq 0$ and $x > 0$, let $F_{n+1}(x) = \int_0^x F_n(t)\,dt$. Evaluate
$$\lim_{n\to\infty} \frac{n! F_n(1)}{\ln n}.$$

11. 2018 A3
Determine the greatest possible value of $\sum_{i=1}^{10} \cos(3x_i)$ for real numbers $x_1, x_2, \ldots, x_{10}$ satisfying $\sum_{i=1}^{10} \cos(x_i) = 0$.

12. 2022 A5
Alice and Bob play a game on a board consisting of one row of 2022 consecutive squares. They take turns placing tiles that cover two adjacent squares, with Alice going first. By rule, a tile must not cover a square that is already covered by another tile. The game ends when no tile can be placed according to this rule. Alice's goal is to maximize the number of uncovered squares when the game ends; Bob's goal is to minimize it. What is the greatest number of uncovered squares that Alice can ensure at the end of the game, no matter how Bob plays?

13. 2012 A6
Let $f(x, y)$ be a continuous, real-valued function on ^{2}. Suppose that, for every rectangular region R of area 1, the double integral of $f(x, y)$ over R equals 0. Must $f(x, y)$ be identically 0?