UNDERGRADUATE SUMMER RESEARCH

TOMMASO BOTTA

Project title: Duality Interfaces in 3d Mirror Symmetry

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Project description: Mirror symmetry in three dimensions is a duality originating in three-dimensional $N = 4$ supersymmetric gauge theories that predicts a deep relationship between two a priori distinct hyperkähler moduli spaces associated to a given theory: the Higgs branch X and the Coulomb branch $X^!$. Mirror symmetry asserts that, for an appropriate dual theory, these two constructions are exchanged, leading to rich correspondences between symplectic singularities, their resolutions, and the representation-theoretic structures attached to them.

The study of these correspondences is an active area of research in both enumerative geometry and representation theory and typically relies on sophisticated tools from each field. However, in sufficiently simple—yet still highly nontrivial—examples, mirror symmetry admits a remarkably explicit description. The proposed project aims to study 3d mirror symmetry from a minimally general and maximally accessible perspective.

The focus will be on the space $X = T^*\mathrm{Gr}(k, n)$, the cotangent bundle of the Grassmannian $\mathrm{Gr}(k, n)$, and on exploring its various manifestations in 3d mirror symmetry. In this setting, abstract statements reduce to nontrivial identities between explicit hypergeometric series, which can be tested both analytically and numerically. In the first phase of the program, students will develop a rigorous mathematical dictionary for mirror symmetry in this context. This includes studying the geometry and cohomology of $\mathrm{Gr}(k, n)$—a prototypical moduli space in algebraic geometry—as well as the theory of hypergeometric series and basic aspects of elliptic functions.

Despite the substantial existing literature on the mirror symmetry of $T^*\mathrm{Gr}(k, n)$, several important aspects remain unexplored. In particular, it is widely expected that the various known incarnations of mirror symmetry descend from the existence of a "duality interface in elliptic cohomology". While the precise mathematical nature of this object is still poorly understood, its existence has been established analytically in the toric case $T^*\mathrm{Gr}(1, n)$ and numerically for general $T^*\mathrm{Gr}(k, n)$. The second phase of the project aims to produce an explicit analytic formula for this duality interface. Success even in the case $k = 2$ would constitute a new and significant result, shedding light on the general picture.

We remark that, although the definition of the interface is abstract in nature, in the present setting the problem becomes extremely concrete: it amounts to determining the unique function satisfying

a specific collection of axiomatic constraints. In particular, the problem can be coded, and analytic solutions can be easily tested.

Overall, we expect this project to provide a general yet challenging introduction to a dynamic and rapidly expanding research field; along the way, students will acquire tools and techniques from geometry, algebra, and representation theory that can be applied transversally across many areas of modern mathematics.

Prerequisites: Prior coursework in algebra (including groups, rings, and modules) and some background in differential geometry, in addition to a solid foundation in standard undergraduate-level mathematics. Knowledge of basic basic algebraic geometry (for example, varieties over the complex numbers) and basic algebraic topology (including homology and cohomology) would be highly beneficial, but is not strictly required.