COLUMBIA | MATH

SAMUEL EILENBERG LECTURES

Professor Robert Bryant

Non-classical PDE in Differential Geometry

Many systems of partial differential equations that arise naturally in differential geometry do not fit easily into the usual paradigms of determined elliptic, hyperbolic, or parabolic systems, and the analysis of such systems has tended to be somewhat ad hoc. A systematic approach to the analysis of such systems, pioneered by Élie Cartan and Erich Kähler and brought to maturity by Masatake Kuranishi (among others), has been available for about 70 years, but the theory is not well-known, which hampers its application in many problems of current interest.

In this lecture series, I will start by discussing a number of interesting problems in differential geometry, such as prescribed curvature or holonomy, isometric deformation and related problems, curvature-homogeneity, calibrated geometry and calibrations, an approach to cluster algebras due to Kontsevich, etc. I will then discuss some tools needed to approach these problems, such as Cartan's generalizations of Lie's fundamental theorems about Lie groups, Cartan characters, and the modern theory of characteristics.

The emphasis will be on understanding the theory through application to interesting examples. Some time will be spent explaining how symbolic calculation software, such as MAPLE, can be used effectively in the analysis of such problems. Familiarity with basic differential geometry and classical PDE will be assumed, but not much beyond this.

Tuesdays @ 2:40 – 3:55pm

Math Hall 520