

Project title: Khovanov homology and quantum error-correcting codes

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Project description: Khovanov homology is a powerful and deep invariant of knots and links. It assigns to every knot (a circle in \mathbb{R}^3), or more generally to every link (a disjoint union of circles in \mathbb{R}^3), a collection of vector spaces $\{V_{i,j}\}_{(i,j) \in \mathbb{Z}^2}$ indexed by a pair of integers. The vector spaces are obtained as the homology of a chain complex which is defined combinatorially starting from a knot or link diagram, and the first key structural property of this theory is that it is a link invariant: two diagrams representing the same link yield isomorphic homology. Khovanov homology has been widely studied since its introduction almost 25 years ago. Applications and generalizations of Khovanov homology continue to be an active and exciting area of research.

Error-correcting codes for quantum computing are crucial to address the fundamental problem of communication in the presence of noise and imperfections. Families of quantum error-correcting codes with desirable properties have been discovered to arise from Khovanov homology. The goal of this project is to explore Khovanov homology and some of its many extensions in order to generate new families of quantum error-correcting codes and to study their properties.

Prerequisites: Linear algebra, modern algebra, and basic algebraic topology. Experience with manifolds is recommended. The relevant background in knot theory, Khovanov homology, and quantum error-correcting codes will be developed during the project.