

COLUMBIA MATH UNDERGRADUATE SUMMER RESEARCH
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GYUJIN OH

Project title: Transcendence and number theory.

Project Description: Most of the named functions (trigonometric functions, log, exponential, Gamma functions, Bessel functions, ...) that are not polynomials are expected to be far from being algebraic. The guiding principle is that a number is not algebraic unless there is a reason for it to be algebraic. For example, one expects that $\cos(1)$ is an irrational number (or even worse a transcendental number), because there seems no way that you could apply the trigonometric identities to $\cos(1)$ to end up with $\cos(\pi/2) = 0$ or $\sin(\pi/2) = 1$. However, proving such claims is more difficult than one might think, and it often involves a fun mix of geometry, algebra, and analysis: one famous example is Apéry's proof of the irrationality of $\zeta(3)$, which can be interpreted nicely using the theory of modular forms.

The aim of the project is to understand the ideas and the curious phenomenon behind the interaction between transcendental functions and number theory/arithmetic geometry.

Prerequisites: The participants should have familiarity with complex analysis and basic modern algebra. Coursework in ordinary differential equations, algebraic number theory, Galois theory, and algebraic curves will be helpful.