COLUMBIA MATH UNDERGRADUATE SUMMER RESEARCH
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NATHAN CHEN

Project title: Low-dimensional varieties and low degree points.

Project Description: The goal of this project is to explore the relationship between several fields of mathematics: algebraic geometry, number theory, and dynamics. In algebraic geometry, the fundamental objects of interest are algebraic varieties. These are the common zero-locus of a collection of polynomials (with coefficients in some field). Over the complex numbers, these often behave like manifolds. However, over number fields they can exhibit very interesting behavior. For instance, the solutions to certain cubic equations give rise to Pythagorean triples – in other words, integers $a, b, c \in \mathbb{Z}$ such that $a^2 + b^2 = c^2$.

One promising avenue of research is the study of low-degree points on algebraic curves and surfaces over a number field. In the one-dimensional case, these are closely tied to the existence of certain low degree maps to another curve. If we replace the curve with a higher dimensional variety such as the symmetric power of the curve or its Jacobian, it is natural to ask what would having low-degree points say about the variety itself?

In another direction, an interesting invariant for a variety is its group of birational self-maps. What happens when this group contains interesting subgroups? The subgroups give interesting Lie algebras which are generated by meromorphic vector fields on the variety, and there are several interesting questions from the point of view of dynamics and complex geometry that can be explored.

Prerequisites: Group theory (Intro to Modern Algebra 1 and 2), point-set topology, and basic complex analysis are strongly recommended. Having some knowledge of algebraic curves will also be helpful.