Fall 2021 Directed Reading Program Colloquium

Topological Proof of the Nielsen–Schreier Theorem

Behshad Jahan Pour

The Nielsen-Schreier theorem, which states that a subgroup of a free group is free, was proved in the early 20th century through algebraic methods. The original proof using algebraic methods is long, and often regarded as unintuitive. In this presentation we will introduce covering spaces, the topological properties of graphs, and outline a topological proof of the theorem.

Law of Large Numbers

Zachary Lihn

There are many theorems in probability concerning sums of a large number of random variables. The most common is the Law of Large Numbers, which says that the average of a sum of independent, identically distributed random variables converges to their mean. In this talk, I will prove a version of the Strong Law of Large Numbers assuming the existence of fourth moments.

Orthonormal Bases of Hilbert Spaces

Shiyang Shen

In this presentation, I will begin with the definition of Hilbert Space and using the L2 space to introduce the properties of inner product. Then I will go through how to construct the orthonormal basis of a general Hilbert Space. Following this, I will use the Fourier Series to explain the orthogonal projection.

Stokes's Theorem

Shuhan Zhang

Stokes's theorem states that the integral of a differential form over the boundary of a orientable manifold is equal to the integral of its exterior derivative over the manifold. It is a generalization of the classical Stokes's Theorem relating 2-dimensional line integrals to 3-dimensional surface integrals. In this talk, I will discuss integration of differential forms over manifolds and present a proof of Stokes's Theorem.

The Perron method for the Laplace equation

Christopher Wang

We prove an existence theorem for the Dirichlet problem for the Laplace equation via the Perron method. In particular, we show that the supremum of a class of so-called subfunctions solves the Laplace equation in a domain and extends continuously to an arbitrary bounded continuous function on the boundary, as long as the boundary of the domain is "regular."

Modular Forms Old and New

Alan Zhao

It is a fundamental result in the theory of modular forms that the space of modular (cusp) forms of level k forms a finite-dimensional complex vector space. In this talk we explain the theory with which these bases may be written as a set of newforms!

Rubinzstein's Topological Link Invariant

Tuan Dolmen

Similar to groups, quandles can be defined as sets with an operation that satisfies certain relations. In a recent paper, it was shown that such structures give rise to a new topological invariant for links. In this talk, I will present the motivation behind quandles, provide a formal definition for quandles, braid actions, and the new topological invariant; prove that this invariant is an honest link invariant, and provide some examples.

A Probabilistic Model for Infection Defaults in Portfolios

Allison Guman

We examine the performance of a typical cash-flow CBO (collateralized bond obligation) transaction, giving attention to the performance of its underlying high-yield-bond portfolio. CBO's effectively concentrate the default risk for equity investors whilst overall reducing risk through diversification. We consider how effective the CBO diversification structure is using an alternative model to the binomial expansion technique (BET) approach used by Moody's where all bonds held are assumed to be independent. In this talk, we describe a 'default infection' model that accounts for interaction between bonds in a given portfolio.

Primes of the form $x^2 + 14y^2$

Destine Lee

A classical result of Euler states that an odd prime is of the form x^2+y^2 if and only if it is congruent to 1 modulo 4. Inspired, we pose the following problem: What are the primes of the form $x^2 + ny^2$ for each positive integer n? Surprisingly enough, this problem of antiquity quickly leads one to twentieth-century mathematics. In this brief presentation, I showcase some of this machinery to examine the case when $n = 14$.

The Convergence of Supermartingales

Harrison Wang

Martingales, due to their many applications to various real-world discrete-time processes, are at the forefront of modern Probability Theory. This talk defines martingales and then goes on to present an interesting result regarding the conditions necessary for the convergence of supermartingales, first proven by Columbia's own Joseph L. Doob.