Title: Knot Floer homology, bordered algebras and double points

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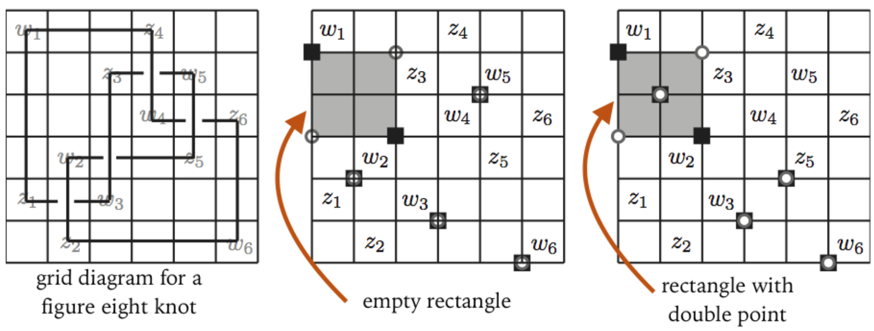
The knot Floer chain complex $\widetilde{CFK}$ is an invariant for knots defined by Ozsváth and Szabó, and independently, Rasmussen. This invariant can be combinatorially computed from a grid diagram representation of a knot. This definition involves counting "empty rectangles" in the grid diagram.

It is natural to elaborate this complex by counting more rectangles. One way of counting more rectangles, has resulted in numerical concordance invariants and lower bounds for 4-ball genus and unknotting number. In another direction, Lipshitz defined a modified version $\widetilde{CFK}'$ of the knot Floer chain complex which counts rectangles with "double points" in addition to the "empty rectangles", see the following figure[1]. It is an open question whether the invariant $\widetilde{CFK}'$ is equivalent to $\widetilde{CFK}$. This project involves investigating the relationship between $\widetilde{CFK}'$ and $\widetilde{CFK}$.



The grid diagram becomes very big, as the number of crossings increase. So a more effective way of computing $\widetilde{CFK}$ is to cut the knot into simple pieces, called *tangles*, and define the invariant for each piece so that $\widetilde{CFK}$ has a nice gluing formula. Two versions of knot Floer homology for tangles have been defined by Petkova-Vértesi and Oszváth-Szabó.

The goals of this REU project are:

(1) Modify the definitions of the tangle invariants (to allow for double points) in order to compute $\widetilde{CFK}'$ via a gluing formula.
(2) Modify the Ozsváth-Szabó program to work for this version.
(3) Can we find an example of a knot such that $\widetilde{CFK}'$ is not equivalent to $\widetilde{CFK}$?

References:

(1) Petkova and Vértesi. https://arxiv.org/abs/1604.08430
(2) R. Lipshitz. Heegaard Floer homology, double points and nice diagrams, 2009. http://pages.uoregon.edu/lipshitz/NiceDoublePoints.pdf
(3) P. Ozsváth and Z. Szabó, An Overview of Knot Floer Homology, 2017. https://arxiv.org/abs/1706.07729.
(4) P. Ozsváth and Z. Szabó, Knot Floer homology calculator. https://web.math.princeton.edu/ szabo/HFKcalc.html

[1] The figure is borrowed from Lipshitz's paper (1).