# MATH W4052 PROBLEM SET 7
# DUE MARCH 7, 2011.

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(1) Let $F(v, w)$ be a bilinear form on a vector space $V$ and $\mathcal{B}$ an ordered basis for $V$. Let $[v]_{\mathcal{B}}$ denote the column vector representation of $v$ with respect to $\mathcal{B} = [e_1, \dots, e_n]$ and $[F]_{\mathcal{B}}$ the matrix representation for $F$ with respect to $\mathcal{B}$. (The $(i, j)^{\text{th}}$ entry of $[F]_{\mathcal{B}}$ is $F(e_i, e_j)$.)

Prove: for any vectors $v, w \in V$

$$F(v, w) = [w]_{\mathcal{B}}^T [F]_{\mathcal{B}} [v]_{\mathcal{B}}.$$

(2) With notation as above, suppose $\mathcal{C} = [f_1, \dots, f_n]$ is another basis for $V$ and $P$ is the change of basis matrix from $\mathcal{C}$ to $\mathcal{B}$ (so the $i^{\text{th}}$ column of $P$ is $[f_i]_{\mathcal{B}}$). Prove:

$$[F]_{\mathcal{C}} = P^T [F]_{\mathcal{B}} P.$$

(3) Cromwell Exercise 6.9.8.
(4) Cromwell Exercise 6.9.9.
(5) Compute the Alexander polynomial for the trefoil and figure 8 knots.
(6) Compute the Alexander polynomial for $T(2, 2n + 1)$. (Use your work for Exercise 6.9.8 and some linear algebra.)

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