MATH W4052 PROBLEM SET 3
DUE FEBRUARY 7, 2011.

INSTRUCTOR: ROBERT LIPSHITZ

(1) Generalize our computation of the fundamental group of the trefoil complement to compute $\pi_1(\mathbb{R}^3 \setminus T_{p,q})$ where $T_{p,q}$ is the (p,q)-torus knot.

(2) Prove that K is 3-colorable if and only if there is a surjective group homomorphism $\pi_1(\mathbb{R}^3 \setminus K) \to D_3$, where D_3, the dihedral group of order 6, is the symmetries of an (equilateral) triangle. (Hint: the dihedral group contains three reflections. A 3-coloring has three colors. Use the Wirtinger presentation.) How does this generalize to n-colorability?

(3) Verify for the Wirtinger presentation for fundamental group of the complement of the figure 8 knot, one of the four relations is redundant with the other three.

(4) (a) Use the Wirtinger presentation to compute the fundamental group of the trefoil from the standard diagram for the trefoil.
    (b) Prove that your answer from the previous part is isomorphic to the group $\langle z, w \mid z^3 = w^2 \rangle$ from class.

(5) Apply Seifert's algorithm to draw Seifert surfaces for the knots 5_1 and 5_2. What are the genera of the surfaces you found?

(6) Is it true that if L is a 2-component link that bounds an embedded annulus then L is the unlink?

(7) Cromwell Exercise 5.9.2.

(8) Cromwell Exercise 5.9.3.

E-mail address: `rl2327@columbia.edu`