# MATH W4052 PROBLEM SET 2
# DUE JANUARY 31, 2011.

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Please keep track of how long this problem set takes you: I'm going to ask, for calibration purposes.

(1) Cromwell Exercise 3.4.
(2) Cromwell Exercise 3.5. (Hint: this is easy from 3.4.)
(3) Cromwell Exercise 3.17.
(4) In Section 2.11, Cromwell gives a rigorous definition of a graph: a set $V$ and a set $E$ of unordered pairs of elements of $V$.
  (a) Give a rigorous definition of a *planar graph* (i.e., a graph embedded in the plane; see page 47 in Cromwell). (Your definition should start something like "A planar graph is a graph $(V, E)$, for each element $v \in V$ a point $f(v) \in \mathbb{R}^2$, and for each pair $\{v_1, v_2\} \in E \dots$".)
  (b) Building on the previous part, give a rigorous definition of a knot diagram. (Suggestion: a knot diagram is a planar graph together with some extra data.)
(5) A knot diagram is *$n$-colorable* if there is a labeling of the strands in the diagram by elements of $\mathbb{Z}/n$ so that at each crossing, if the over-strand is labeled $a$ and the two under-strands are labeled $b$ and $c$ then

$$2a \equiv b + c \pmod{n}$$

(and not all strands are colored by the same number).
  (a) Verify that $n$-colorability depends only on the knot type, not the particular diagram, by checking it's unchanged by Reidemeister moves.
  (b) Explain that the unknot is not $n$-colorable for any $n > 1$. (Hint: this is trivial.)
  (c) Show that the Figure 8 knot is 5-colorable. (So, the Figure 8 knot is not the unknot.)

(This exercise is similar to Lickorish's Exercise 9 in Chapter 1.)
(6) Let $K$ be a knot in $\mathbb{R}^3$. Recall that $S^3$ is the one-point compactification of $\mathbb{R}^3$, so we can view $K$ as sitting in $S^3$. Prove that $\pi_1(\mathbb{R}^3 \setminus K) \cong \pi_1(S^3 \setminus K)$.
(7) Generalize our computation of the fundamental group of the trefoil complement to compute $\pi_1(\mathbb{R}^3 \setminus T_{p,q})$ where $T_{p,q}$ is the $(p, q)$-torus knot.

Also, read through the rest of the exercises in Cromwell, Chapter 3.

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