MATH G4307 PROBLEM SET 6
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Exercises to turn in:

(E1) Hatcher 2.1.20 (p. 132).
(E2) Hatcher 2.1.21 (p. 132).
(E3) Hatcher 2.1.22 (p. 132).
(E4) Use the Mayer-Vietoris sequence and / or the long exact sequence for a pair to compute:
  (a) $H_*(T^2)$, the torus.
  (b) $H_*(T^2 \setminus \mathbb{D}^2)$.
  (c) $H_*(T^2 \setminus (\mathbb{D}^2 \amalg \mathbb{D}^2))$.
  (d) $H_*(\Sigma_g)$, the orientable surface of genus g.
  (e) $H_*(\mathbb{R}P^2)$, the real projective plane.
(E5) Hatcher 2.2.29 (p. 158).
(E6) Hatcher 2.2.34 (p. 158): "Derive the long exact sequence of a pair (X, A) from the Mayer-Vietoris sequence applied to $X \cup CA$, where CA is the cone on A. [We showed after the proof of Proposition 2.22 that $H_n(X, A) \cong \widetilde{H}_n(X \cup CA)$ for all n.]"

  (This was removed from the online version of the book, because the proof that $H_n(X, A) \cong \widetilde{H}_n(X \cup CA)$ uses the long exact sequence for the pair. Ignore that point, or view this $\cong$ as a definition of the left hand side.)
(E7) Hatcher 2.2.35 (p. 158).

Problems to think about but not turn in:

(P1) Spend some time with the snake lemma. Prove it without referring to the book. Next, go through the proof again keeping track of the topology, in terms of the long exact sequence for a pair.
(P2) In Exercise (E4), try doing as many of the computations as you can using only the Mayer-Vietoris sequence, and using only the long exact sequence for a pair (and excision).
(P3) Recall that we defined $H^1(X) = [X, S^1]$. Let's define $H^0(X)$ to be $[X, \mathbb{Z}]$, where we give $\mathbb{Z}$ the discrete topology. The results we've proved for homology—e.g., the long exact sequence for a pair, the Mayer-Vietoris sequence, excision—have analogues for cohomology (but all the maps go the other way). Formulate and prove these for the H^0 and H^1 parts of cohomology.

  (Hint: let $H^1(X, A) = [(X, A), (S^1, 1)]$.)
(P4) Read through the remaining problems in this section, and do any that seem difficult, surprising or interesting.

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