MATH W4051 PROBLEM SET 7
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(1) Munkres 52.1
(2) Munkres 55.2
(3) Munkres 55.4 parts (a)–(d)
(4) Does every continuous map $S^2 \to S^2$ have a fixed point? If so, prove it. If not, give a counterexample, and see if you can find a more restrictive statement which you think is true.
(5) The fundamental group of products...
  (a) Let X and Y be path-connected spaces. Prove that $\pi_1(X \times Y) = \pi_1(X) \times \pi_1(Y)$.
  (b) Conclude that T^2 is not homeomorphic to S^2.
  (c) What is $\pi_1(\mathbb{R}^2 \setminus 0)$?
  (d) Show that T^3, $S^1 \times S^2$ and S^3 are all distinct (i.e., no pair of them is homeomorphic). (Recall that $T^3 = S^1 \times S^1 \times S^1$.)
(6) (From Hatcher): Show that composition of paths has the following cancellation property: Let γ_0, γ_1 be paths from p to q and η_0, η_1 paths from q to r. Suppose that $\gamma_0 * \eta_0 \sim \gamma_1 * \eta_1$ (rel endpoints) and $\eta_0 \sim \eta_1$ (rel endpoints). Then $\gamma_0 \sim \gamma_1$ (rel endpoints).

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