Holomorphic curves and stable homotopy theory

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Let $M(X)$ denote the moduli space of (nodal, stable, genus g) curves holomorphic curves in X. Invariants (e.g. Gromov-Witten) can be extracted from two ingredients: (Parts of) the cohomology ring $H^*(M(X))$, and the fundamental class $H^*(M(X)) \to \mathbb{Q}$. I will describe a space $F(X)$ and a natural map $u : M(X) \to F(X)$. From the point of view of stable homotopy theory, the definition of the space $F(X)$ is a rather natural construction, and in particular the rational cohomology ring of $F(X)$ can be easily and explicitly described. All relevant cohomology classes in $M(X)$ arise as pull back from classes in $F(X)$. This framework collects all the "homotopy theoretic" information into one object, and all the "analytic" information is encoded in the fundamental class $H^*(F(X)) \to \mathbb{Q}$. In the case where X is a point (my talk will focus on this case), the fundamental class "is" the power series determined by Kontsevich's theorem (Witten's conjecture).This is joint work with Ya. Eliashberg.

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