GEOMETRIC ASPECTS OF THE DIRICHLET PROBLEM

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ABSTRACT

I will discuss an approach to solving the Dirichlet problem for a variety of nonlinear partial differential equations which arise in geometry. In this approach, inequalities of the form $f(x, u, Du, D^2u) \geq 0$ are replaced by subsets of the 2-jet space. Nothing is lost from the original theory, but many analytical properties can be seen clearly from the geometry of these subsets. In particular, an illuminating duality emerges which is used, among other things, for setting the boundary geometry necessary for solving the problem.

Original motivation for this work came from the fact that manifolds with geometric structure often carry large and useful families of non-standard "subharmonic" functions. For example, any almost complex manifold with hermitian metric carries plurisubharmonic functions. It also carries "Lagrangian subharmonic functions" whose restrictions to minimal Lagrangian submanifolds are subharmonic. Each Calabi-Yau manifold carries several interesting families. In fact every manifold with a calibration φ carries "φ-plurisubharmonic functions" which are subharmonic on all calibrated submanifolds. In all cases the extremals in these families, the *harmonic functions*, are interesting and often satisfy a basic non-linear second-order equation.

I will discuss the Dirichlet Problem for such harmonic functions on bounded domains in a riemannian manifold. Existence and uniqueness will be established for these and many other quite general second-order equations. The results hold for all continuous boundary data subject to a geometric *F-convexity* of the boundary, defined entirely in terms of the equation F.

Examples include all branches of the homogeneous Monge-Ampère equation over $\mathbf{R}$, $\mathbf{C}$ and $\mathbf{H}$, and all branches of the special lagrangian potential equation.

This is joint work with Reese Harvey.