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SET THEORY

Contents

Section 1. Introduction

We need some set theory every now and then. We use Zermelo-Fraenkel set theory with the axiom of choice as described in [Kun83]. Since we are talking about potentially large objects (categories and categories of categories) we should be carefull.

Section 2. Everything is a set

Explain how everything is a set.

Subsection 2.1. The hierarchy of sets. A set T is transitive if $x \in T$ implies $x \subset T$. A set α is an ordinal if it is transitive and wellordered by $\in$. We define, by transfinite induction, $V_0 = \emptyset$, $V_{\alpha+1} = P(V_\alpha)$, and for a limit ordinal α,

$$V_\alpha = \bigcup_{\beta<\alpha} V_\beta.$$

Every set is contained in one of the V_α.

Subsection 2.2. Everything is is contained in some ordinal. The title says it all.

Section 3. Reflection principle

This explains how we deal with set theoretical difficulties.

Subsection 3.1. Statement of the theorem. Let $\phi(x_1, \ldots, x_n)$ be a formula of set theory. Let V be a set. The formula $V \models \phi(x_1, \ldots, x_n)$ is the formula obtained from $\phi(x_1, \ldots, x_n)$ replacing all the $\forall x$ and $\exists x$ by $\forall x \in S$ and $\exists x \in S$. (So the formula $\phi(x_1, x_2) = \exists x, (x \in x_1 \wedge x \in x_2)$ is turned into $S \models \phi(x_1, x_2) = \exists x, ((x \in S) \wedge (x \in x_1 \wedge x \in x_2))$.

Theorem 3.1.1. *Let $\phi(x_1, \ldots, x_n)$ be a formula of set theory, and let T be a set. There exists an α such that $\forall x_1, \ldots, x_n \in V_\alpha$,*

$$V_\alpha \models \phi(x_1, \ldots, x_n) \Leftrightarrow \phi(x_1, \ldots, x_n).$$

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1. (1) visit the next section: Categories, Section 1, or
2. (2) go back to the table of contents: `index.html#contents`.

References

[Kun83] Kenneth Kunen. *Set Theory*. Elsevier Science, 1983.