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CONVENTIONS USED IN THE ALGEBRAIC STACKS DOCUMENTS

1. Comments

The philosophy behind the conventions used in writing these documents is to choose those conventions that work. In the end the precise choices we make here probably do not make a big difference in the resulting theory of algebraic stacks of finite type over the integers (or over a field, or over an excellent Noetherian ring). Also, perhaps the higher level thoery is flexible enough so as to allow for different choices here, but still we have to choose one.

2. Set theory

We use Zermelo-Fraenkel set theory with the axiom of choice.

2.1. Additional.

A class is not a set and hence is not described by Zermelo-Fraenkel set theory. You can think of a class (such as the class of all sets, or the class of rings) as the collection of sets satisfying a rule. Or if you prefer you can use von Neumann-Bernays-Gödel set theory which is a conservative extension of Zermelo-Fraenkel set theory.

3. Categories

A category C consists of a class of objects, and for each pair of objects a set of morphisms between them. A category is called small if the class of objects is in fact a set. There are no set theoretic difficulties in defining functors and natural transformations of functors. We will call a natural transformation of functors $F \to G$ simply a morphism of functors.

3.1. Equivalences of categories.

Two categories A and B are said to be equivalent if there exist functors $F : A \to B$ and $G : B \to A$, and isomorphisms $\mathrm{id}_B \to F \circ G$ and $\mathrm{id}_A \to G \circ F$. Recall that a functor $F : A \to B$ is fully faithfull if for any objects X, Y of $\mathrm{Ob}(A)$ the map $F : \mathrm{Mor}_A(X, Y) \to \mathrm{Mor}(F(X), F(Y))$ is bijective. Recall that F is called essentially surjective if for any object $Z \in \mathrm{Ob}(B)$ there exists an object $X \in \mathrm{Ob}(A)$ such that $F(X)$ is isomorphic to Z.

The following lemma will be used repeatedly.

Lemma 3.1.1. *Suppose that $F_i : A_i \to B$, $i = 1, 2$ are two functors from small categories to a category B. Suppose that F_1 and F_2 are fully faithfull and essentially surjective. Then A_1 and A_2 are equivalent.*

Proof. Obvious. □

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4. Algebra

In these notes a ring is a commutative ring with a 1. Hence the category of rings has an initial object $\mathbf{Z}$ and a final object $\{0\}$ (this is the unique ring where $1 = 0$). Modules are assumed unitary.

Here are some references for this section: [Eis95].

4.1. **Properties of modules.** This subsection recalls the definitions of some of the more common properties of modules over rings. Let R be a ring and let M be an R-module.

4.2. **Properties of ring maps.** This section lists various properties of ring maps. Let $R \to S$ be a homomorphism of rings.

References

[Eis95] David Eisenbud. *Commutative Algebra*, volume 150 of *Graduate Texts in Mathematics*. Springer-Verlag, 1995.