MINERVA RESEARCH FOUNDATION LECTURES
Department of Mathematics
Columbia University

Professor Grigori Olshanski

Spring 2012
Fridays 2:15-4:00 PM
Room 312 Mathematics

First Meeting: Friday January 20, 2012

DETERMINANTAL PROCESSES AND RELATED TOPICS

Determinantal processes form a special class of random point processes that are on the next level of complexity after Poisson processes. Numerous examples of determinantal processes emerge in different domains of mathematics and mathematical physics: probability theory, random matrix theory, tiling models, algebraic combinatorics, representation theory; the subject is also related to classical analysis and special functions.

The theory of determinantal processes is a relatively new and rapidly developing subject. The goal of the course is to give a comprehensive introduction to this theory and to review some of its most recent achievements.

The course is accessible to graduate students. Prerequisites: Basic Real Analysis and Probability Theory (measure and integration, Markov chains, elements of Markov processes); Linear Algebra; some familiarity with basic notions related to Banach and Hilbert space operators would also be desirable.

Brief course description

Elements of General Theory: Random point processes. Janossy functions and correlation functions. Determinantal point processes. Correlation kernels. Macchi-Soshnikov theorem.

Basic Examples: Sine-kernel process and discrete sine-kernel process. Orthogonal polynomials, Christoffel-Darboux kernel. N-particle ensembles: orthogonal polynomial ensembles and biorthogonal ensembles. Large-N limit transitions.

Random Matrices: Random matrices and random spectra. Dyson's circular unitary ensemble CUE_N, convergence to the sine-kernel process. Cayley transform relating unitary and Hermitian matrices. Infinite-dimensional random matrices.

Random Partitions: Partitions and Young diagrams. Combinatorics of the Young graph. Probability measures on Young diagrams: z-measures and Plancherel measure. Ulam's problem. Baik-Deift-Johansson theorem and its generalization.

Space-time Determinantal Processes: Dyson's Brownian motion model. Eynard–Mehta theorem. Non-intersecting path models. General structure of space-time correlation kernels.

Markov Chains on Partitions related to z-measures; Plancherel Measure: Dynamical Meixner ensemble. Analytic continuation in dimension. Space-time hypergeometric kernel. Viennot's geometric version of Robinson-Schensted correspondence. From Poisson process in a quarter-plane to Plancherel dynamics.

Projective Limit Construction of infinitely-many-particle Feller dynamics: Branching graphs and Martin boundaries. Commutative diagrams of Markov kernels. The projective limit construction of Markov dynamics.

Literature

The material of the course is based mainly on recent journal publications and preprints. A few expository papers listed below can give an idea of the subject but should not be regarded as main references:

A. Borodin, *Determinantal point processes*, In: The Oxford Handbook of Random Matrix Theory, Chapter 11. Oxford University Press, 2011; arXiv:0911.1153.

A. Borodin and G. Olshanski, *Z-Measures on partitions, Robinson-Schensted-Knuth correspondence, and $=2$ ensembles*. In: Random matrix models and their applications (P. M. Bleher and A. R. Its, eds). MSRI Publications 40, Cambridge Univ. Press, 2001, pp. 71–94; arXiv:math/9905189.

A. Borodin and G. Olshanski, *Representation theory and random point processes*. In: A. Laptev (ed.), European congress of mathematics. Stockholm, Sweden, June 27–July 2, 2004. Zürich: European Mathematical Society, 2005, pp. 73–94; math/0409333.

J. B. Hough, M. Krishnapur, Y. Peres and B. Virag, *Determinantal processes and independence*. Probability Surveys **3** (2006) 206–229; arXiv:math/0503110.

W. König, *Orthogonal polynomial ensembles in probability theory*. Probability Surveys **2** (2005) 385–447; arXiv:math/0403090.

R. Lyons, *Determinantal probability measures*. Publ. Math. Inst. Hautes Etudes Sci. **98** (2003) 167–212; arXiv:math/0204325.

G. Olshanski, *Random permutations and related topics*. In: The Oxford Handbook of Random Matrix Theory, Chapter 25. Oxford University Press, 2011; arXiv:1104.1266.

A. Soshnikov, Determinantal random point fields, Russian Math. Surveys **55** (2000) 923–975; arXiv: math/0002099.