COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

DEPARTMENT OF MATHEMATICS

Samuel Eilenberg Lectures

Fall 2010

Spencer Bloch

University of Chicago

"Algebraic Geometry and Quantum Field Theory"

The correlation functions arising in QFT have perturbative expansions which are infinite series indexed by graphs. The coefficient associated to a graph is a period in the sense of algebraic geometry, and physicists spend a great deal of time computing these periods. The development of the Connes-Kreimer hopf algebra of graphs has given considerable impetus to the qualitative study of these periods. When, e.g., are they multiple zeta numbers? In addition, the period integrals frequently diverge, and considerable effort has gone into developing convergence methods to make sense of the coefficients. These lectures will focus on techniques from algebraic geometry which I hope can be used to give the physicists' work a rigorous mathematical foundation. Among the results I want to touch upon is the theory of determinantal varieties and its generalization to quaternionic pfaffian varieties. These furnish the basic motives for the physical periods. Then I will explain the Connes-Kreimer hopf algebra and relate its structure to algebraic geometry. The end result is to interpret Green functions as 1-parameter subgroups on the spectrum of the Connes-Kreimer hopf algebra. Further topics include the recent example of Brown and Schnetz of a graph where the period is related to a Kummer surface associated to a product of a CM elliptic curve with itself, and a quantitative calculation for 1-loop graphs with arbitrary masses and momenta. Although this work is "informed" by algebraic geometry, I will not have to use any elaborate algebraic techniques beyond rational differential forms and their linear algebra.

FRIDAYS, 3:30PM

312 Mathematics Hall
2990 Broadway at 117th St
New York City

First lecture:
September 17th