

# Lie Superalgebras and Categorification Seminar Outline

Spring 2023

## 1 Lie Superalgebras

- (1) **Lie Superalgebras: Fundamentals**
- (2) **Lie Superalgebras: Representations**
  - (a) Follow [1] Section 1.5.2-1.5.3.
  - (b) Follow [1] Section 2.1.1.
  - (c) Follow [1] Section 2.2.1-2.2.2
- (3) **Lie Superalgebras: Character Formulas**
  - (a) Follow [1] Section 2.2.3.
  - (b) Cover [1] Lemma 2.24 and state Theorem 2.26.
  - (c) Follow [1] Section 2.2.6-2.2.7 (only the  $\mathfrak{gl}(m|n)$  parts).

## 2 Main Categorification Results

- (4) **Categorification of quantum  $\mathfrak{gl}(1|2)$  I**  
[4, How to categorify one-half of quantum  $\mathfrak{gl}(1|2)$ ]
- (5) **Categorification of quantum  $\mathfrak{gl}(1|2)$  II**  
[4, How to categorify one-half of quantum  $\mathfrak{gl}(1|2)$ ]  
[5, A categorification of the positive half of quantum  $\mathfrak{gl}(m|1)$ ](Maybe only 10 minutes on this?)
- (6) ???
- (7) **Categorification of Clifford algebras I**  
[9, A diagrammatic categorification of a Clifford algebra]
- (8) **Categorification of Clifford algebras II**  
[9, A diagrammatic categorification of a Clifford algebra]  
[8, A Categorification of  $U_q(\mathfrak{sl}(1|1))$  as an algebra]

## 3 More Categorification Results

- (9) **Crystal Basis for  $\mathfrak{gl}(m|1)$**   
[2, Canonical bases for the quantum enveloping algebra of  $\mathfrak{gl}(m|1)$  and its modules]
- (10) **MOY Calculus for  $U_q(\mathfrak{gl}(1|1))$**   
[3, A generators and relations description of a representation category of  $U_q(\mathfrak{gl}(1|1))$ ]
- (11) **Categorification of tensor products of  $\mathbb{C}^{1|1}$**   
[7, Categorification of tensor powers of the vector representation of  $U_q(\mathfrak{gl}(1|1))$ ]

- (12) **Ozsváth-Szabó bordered algebras, Satori algebras and Category  $\mathcal{O}$**  [6, Ozsváth-Szabó bordered algebras and subquotients of category  $\mathcal{O}$ ]

## References

- [1] S.-J. Cheng and W. Wang. Dualities and representations of Lie superalgebras. Vol. 144. Graduate Studies in Mathematics. American Mathematical Society, Providence, RI, 2012, pp. xviii+302.
- [2] S. Clark. Canonical bases for the quantum enveloping algebra of  $\mathfrak{gl}(m|1)$  and its modules. 2016. arXiv: [1605.04266](https://arxiv.org/abs/1605.04266).
- [3] J. Grant. “A generators and relations description of a representation category of  $U_q(\mathfrak{gl}(1|1))$ ”. In: Algebr. Geom. Topol. 16.1 (2016), pp. 509–539.
- [4] M. Khovanov. “How to categorify one-half of quantum  $\mathfrak{gl}(1|2)$ ”. In: Knots in Poland III. Part III. Vol. 103. Banach Center Publ. Polish Acad. Sci. Inst. Math., Warsaw, 2014, pp. 211–232.
- [5] M. Khovanov and J. Sussan. “A categorification of the positive half of quantum  $\mathfrak{gl}(m|1)$ ”. In: Trans. Amer. Math. Soc. 369.3 (2017), pp. 1627–1664.
- [6] A. D. Lauda and A. Manion. “Ozsváth-Szabó bordered algebras and subquotients of category  $\mathcal{O}$ ”. In: Adv. Math. 376 (2021), Paper No. 107455, 59.
- [7] A. Sartori. “Categorification of tensor powers of the vector representation of  $U_q(\mathfrak{gl}(1|1))$ ”. In: Selecta Math. (N.S.) 22.2 (2016), pp. 669–734.
- [8] Y. Tian. “A CATEGORIFICATION OF  $U_q(\mathfrak{sl}(1|1))$  AS AN ALGEBRA”. In: J. Symplectic Geom. 14.2 (2016), pp. 541–585.
- [9] Y. Tian. “A diagrammatic categorification of a Clifford algebra”. In: Int. Math. Res. Not. IMRN 21 (2015), pp. 10872–10928.