9. Hyperbolic Geometry

These exercises must be carried out using NonEuclid (link on the website). Do as many as you can.

For each exercise, select "Clear All" under "Select Measurement or Modification"

1. Lines

(a) Two lines are parallel if they don't intersect inside the disk. Construct three lines (using "Line")

Call them ℓ_1, ℓ_2, ℓ_3. They must satisfy:

- ℓ_1, ℓ_2 intersect at a single point.
- ℓ_3 is parallel to both ℓ_1 and ℓ_2.

(b) Construct two lines (using "Line"). Construct their intersection point (using "Intersection point" and its instructions.)

Next, measure the 4 angles at the point of intersection (see the instructions under "Measure angle")

Finally, move the points you used to define the lines in order to make the 4 angles 90°.

2. Perpendicular bisector.

(a) Draw a line segment, with endpoints A and B.

(b) Draw a circle with center A and passing through B.
Draw a circle with center B and passing through A.

(c) Draw a line through the intersection points of the 2 circles.

(d) Check that the line and segment are perpendicular (measure the angle).

(More exercises on the next page).

3. Triangles

(a) Construct a triangle (using line segments) with angles adding up to 100°.

Then one with angles adding up to $<05^\circ$.

(b) Construct a triangle and draw perpendicular bisectors to each side (Feel free to hide the auxiliary circles). By moving the points around, convince yourselves that the three bisectors intersect in a single point. Finally find the circle that passes through the vertices of the triangle.

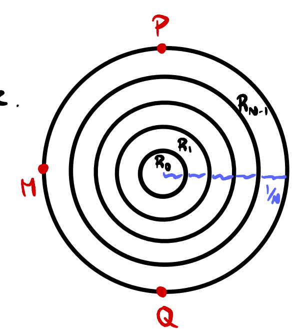
(c) Construct a triangle $\widehat{ABC}$. Choose a point D on the segment BC. Finally measure the triangles ("Measure triangles") $\widehat{ABC}$, $\widehat{ABD}$, $\widehat{ADC}$. Let $A(\text{triangle}) = 180^\circ - \text{angle sum}$. Check that $A(\widehat{ABC}) = A(\widehat{ABD}) + A(\widehat{ADC})$. Call A the "area" of the triangle.

4. Areas

(a) Draw two triangles with equal side lengths (in different parts of the disk). Check that their angles are equal. Is this true in Euclidean geometry?

(b) Draw two triangles with equal angles (in different parts of the disk). Check that their side lengths are equal. Is this true in Euclidean geometry?

(c) Conclude from this and 3.c) that the notion of area we defined is reasonable.



5. (Harder) Towards differential geometry.

Consider the unit disk $\{(x,y)\in\mathbb{R}^2 : x^2+y^2=1\}$ divided into annular regions $R_0, \dots, R_{N-1}$ each of width $\frac{1}{N}$.

Define the norm $\|(x,y)\| = \sqrt{x^2+y^2}$. Define a distance on each R_k given by $d(p,q) = \dfrac{\|p-q\|}{1-\left(\frac{k}{N}\right)^2}$.

Sketch what you imagine is the shortest path between P and Q. Do the same for P and M.

How would you formalize this setup "as $N\to\infty$". For instance, how would you compute the length of a path?

10. Hyperbolic Knot Theory

1. Go over to Hyperrogue to experience the hyperbolic plane "from a local perspective".
2. Go over to Hypernom to experience hyperbolic 3D space "from a local perspective".

   (Use the arrow keys + WASD)

3. Open SnapPy (install it if you haven't yet) and input: M = Manifold()

The link editor will pop up. Draw any link and click on Tools > Send to SnapPy. Verify that your link is hyperbolic by computing its volume via M.volume(). (If it isn't, it will return 0). Then explore the hyperbolic geometry of the complement of your link using

- M.dirichlet_domain().view()
- M.inside_view() (Use the arrow keys + WASD)