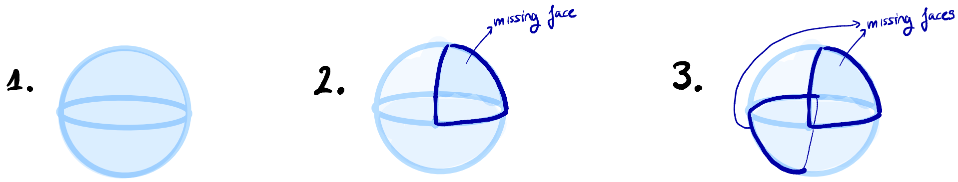
5. The topology of surfaces

1. Reconstruct the deflated polyhedron for the sphere that uses 6 vertices.
Count the number of edges and vertices in the deflated polyhedra for the surfaces below:



Notice a pattern in the Euler characteristics and conjecture a relation between $\chi(S)$ and $\chi(S')$, where S is any surface and S' is S minus a disk.
Convince yourselves that your result is always true.

2. Define the connected sum of two surfaces as follows: remove a disk from each and glue them along the boundary:

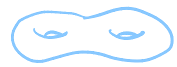
1. Prove that $T_{i,0} \# T_{j,0} = T_{i+j,0}$ for any $i,j \geq 0$.
2. Prove that given two surfaces S_1 and S_2, $\chi(S_1 \# S_2) = \chi(S_1) + \chi(S_2) - 2$.
   (Hint: remove a triangle from each associated polyhedron)
3. Use 1 and 2 to give a formula for $\chi(T_{g,0})$ in terms of $g \geq 0$.
   (You may use the fact that $\chi(T_{1,0}) = 0$).

3. Combine your results from 1 and 2 to give a formula for $\chi(T_{g,b})$ in terms of g and b.

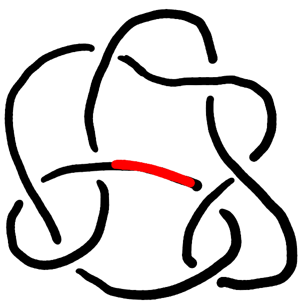
4. (Optional, hard) Find a surface without boundary, not necessarily "embedded" in $\mathbb{R}^3$ and with Euler characteristic 1.

6. Topology in higher dimensions

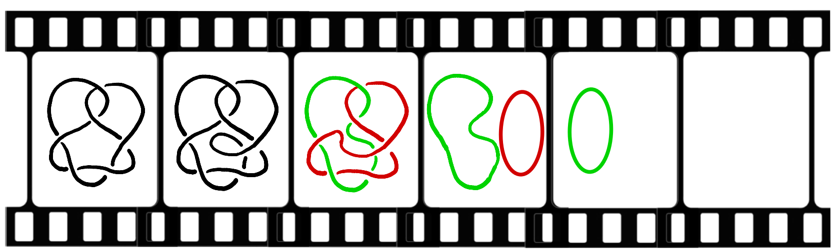
1. If a sphere is the analog of a circle one dimension up, what's the analog of a circle one dimension down?

2. Draw the 2D-movie for the double torus 

3. Show that the following knot is slice



Reminder:



4. Explain why a 2-sphere in $\mathbb{R}^4$ does not separate $\mathbb{R}^4$ into two regions like it does in $\mathbb{R}^3$. Describe a manifold that does.

5. (Harder) Can you find examples of 3-manifolds that are not the 3-sphere?