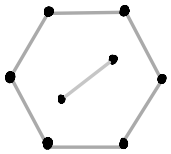
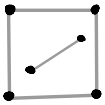
Note: If an exercise has a circle around it:
- If your group's name is a number, solve a)
- If your group's name is a letter, solve b)
- You will solve c) together with the other group.

3. Combinatorics and our first invariant



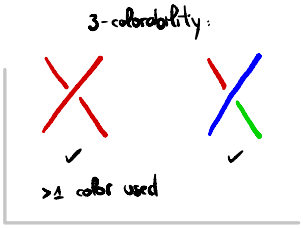
(1.) a) How many ways are there to color the edges of the graph with the colors 0 and 1, so that no two edges of the same color are adjacent?



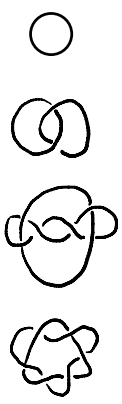
b) How many ways are there to color the edges of the graph with the colors 0 and 1, so that no two edges of the same color are adjacent?

c) Convince yourselves that a graph containing a polygon with an odd number of sides is <u>not</u> colorable using 2 colors.

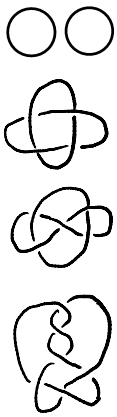
(2.) Determine whether the following links are 3-colorable



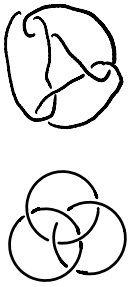
a)



b)



c)



3. Find a map (a partition of a rectangle into regions) that is 4-colorable but not 3-colorable.

4. Is it possible for a group of 5 people to satisfy property P?

<u>Property P</u>: no group of 3 people are all friends,
and in every group of 3 people at least two of them are friends.

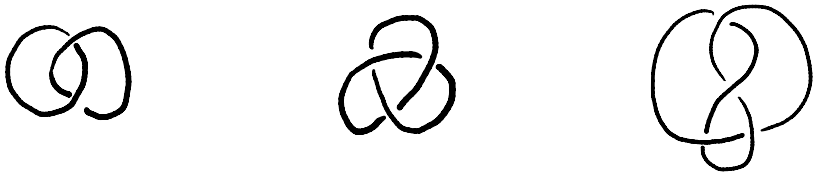
5. (Hard) Define a notion of 4-colorability for links and show that it is a link invariant.

4. Modular arithmetic and n-colorability

1. Reduce the following numbers modulo n:

- $2+3\cdot 5 \pmod{11}$
- $7^4 \pmod 8$
- $2^{10} + 6\cdot 9^{10} \pmod 7$
- $a^5 - a \pmod 5$ for $a = 0, 1, 2, -1, -2$.

2. Determine whether the following links are 3-colorable and/or 5-colorable.



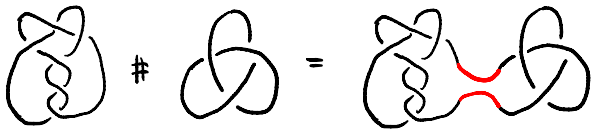
Use this to prove that no two of them are isotopic.

3. Input the links in exercise 2 into Sage by using SnapPy in order to obtain their PD codes. Then use the commands `.is_colorable(3)`, `.is_colorable(5)` to double-check your answers to 2.

Reminder:

```
import snappy
PD=[(4,1,3,2),(2,3,1,4)];
L_snappy=snappy.Link(PD);
L=L_snappy.sage_link();
L.plot()
```

4. Define the connected sum of two knots as follows.



Prove that if K_1 is p-colorable and K_2 is q-colorable then $K_1 \# K_2$ is both p- and q-colorable.

5. (Hard) A knot is called invisible if it is <u>not</u> p-colorable for any prime p. Can you find any invisible knots? Can you find infintely many?