Intro to Abstract Math (via Knots)

SHP Spring 2022

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Please fill out the form!

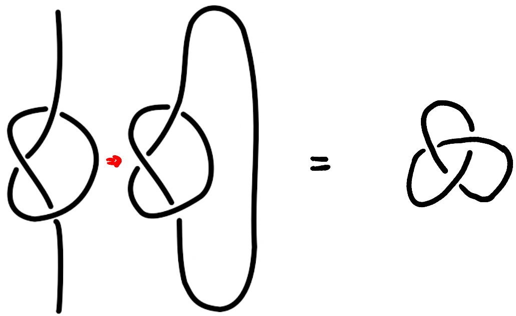
What is a Knot?

1. Take a string

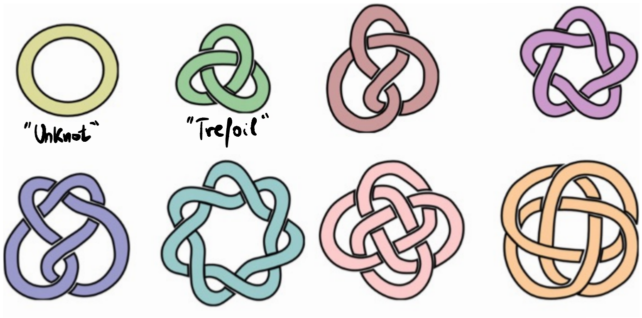
2. Knot it however you want.

3. Fuse the ends together.

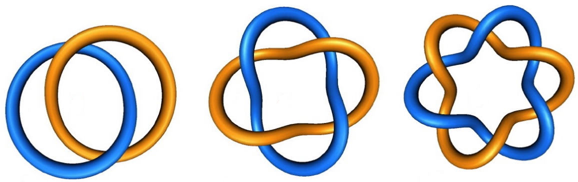
Example:

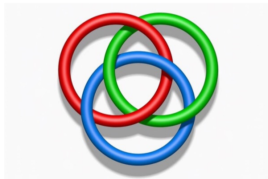


More examples:



If we knot several strings together, we get a link:

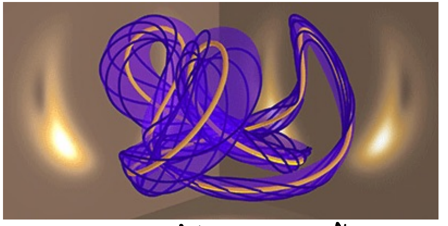




They can get complicated:

Mathematical curiosity?

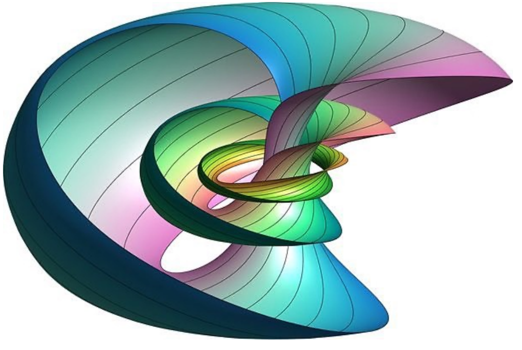
- In electromagnetism:



path in the electromagnetic field

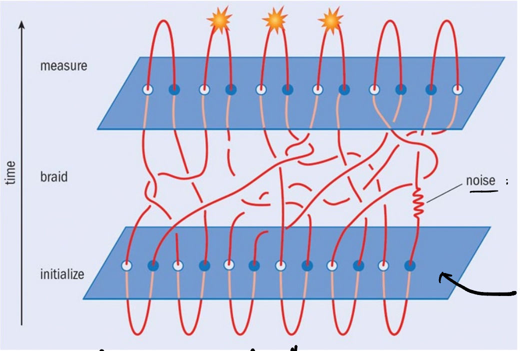
- In mathematics:

all possible "3D-spaces" can be constructed from knots



A 3-manifold in the making.

- In quantum physics

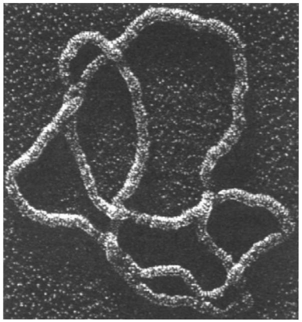


: does not affect the computation as long as the knot is the same.

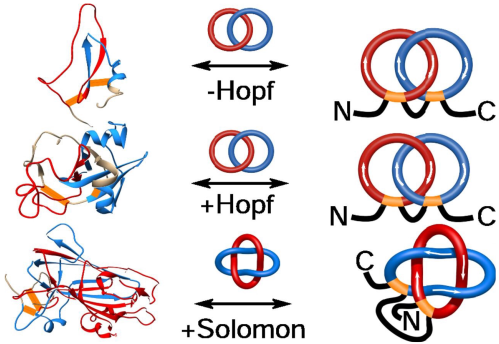
qubits

Fractional quantum Hall effect

- In biology:



Knotted DNA strands



The knottedness of proteins affects their functions.

Good news: no need to know any of those things to do knot theory!

Goals of the course:

1. Understand knots from a mathematical point of view.
   - When are two links "the same"?
   - How can I tell two links apart?

2. Explore connections with other fields in math

   (and use knot theory as an excuse to explore them)

3. Learn how to use software to do math

4. Get a feel for what math research is like.

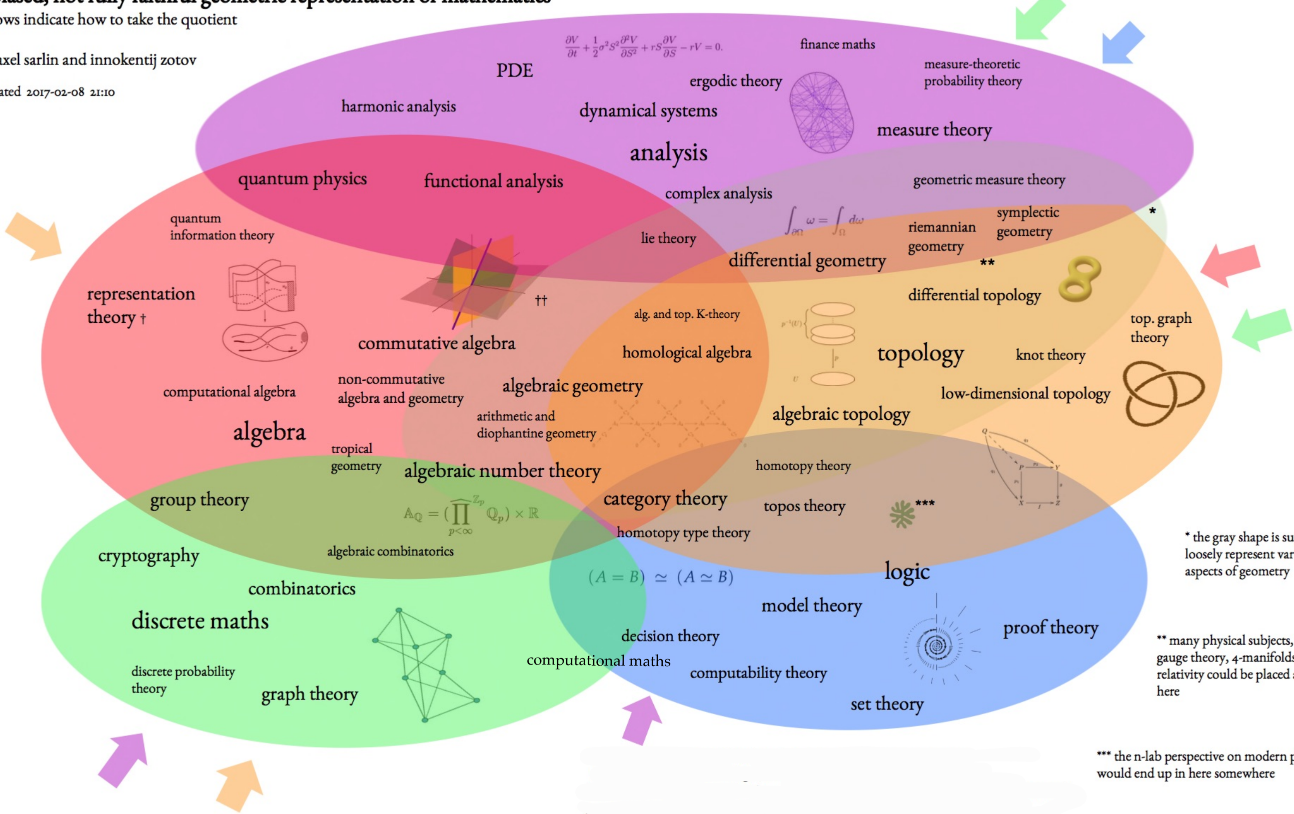
Map of pure mathematics:

a biased, not fully faithful geometric representation of mathematics

arrows indicate how to take the quotient

by axel sarlin and innokentij zotov

updated 2017-02-08 21:10

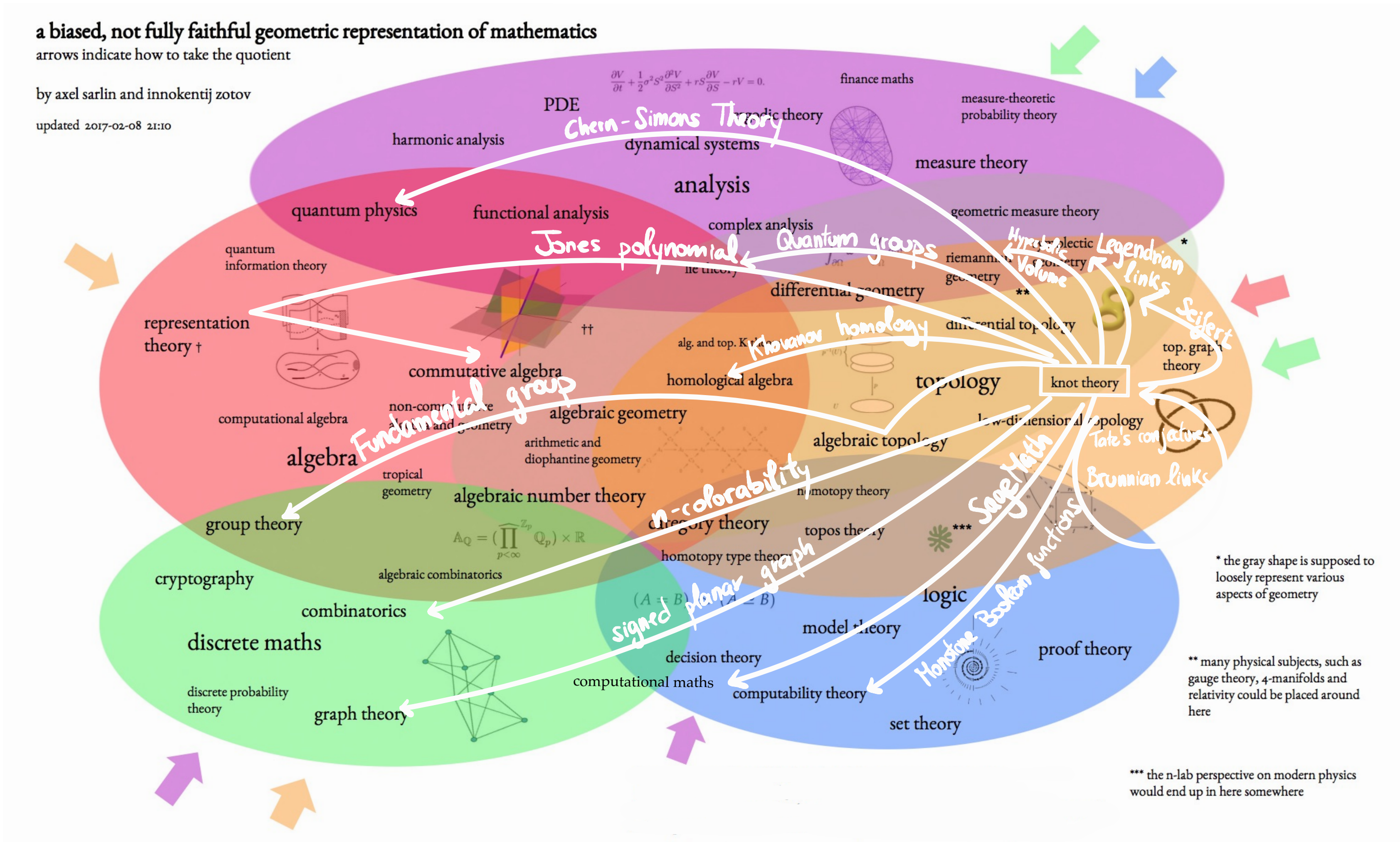


* the gray shape is supposed to loosely represent various aspects of geometry

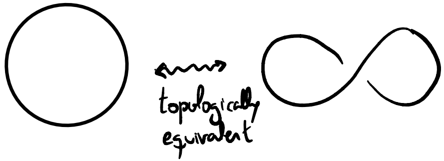
** many physical subjects, such as gauge theory, 4-manifolds and relativity could be placed around here

*** the n-lab perspective on modern physics would end up in here somewhere

Connections to knot theory :



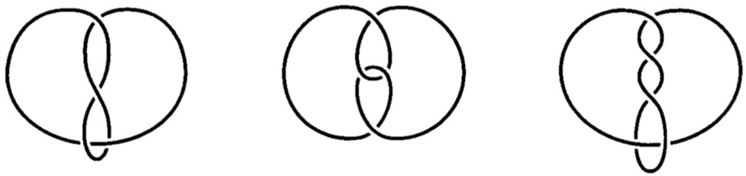
0. The unknot



https://miro.medium.com/max/600/1*z8xAu-m3jzDqfxjrgohjnw.gif

Definition (link diagram): a projection of a link onto the plane so that we can tell under/overcrossings.

Examples:



Poll: is this a link diagram for the unknot?

Q?

Poll: which of these can be manipulated into trefoils?

a.

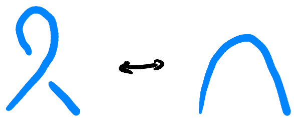
b.

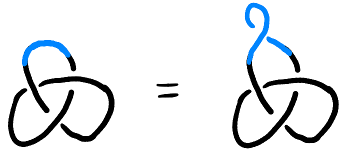
c.

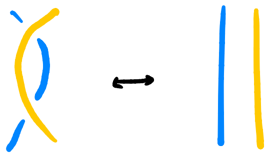
d.

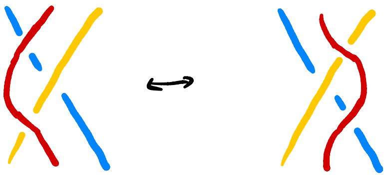
http://sites.oglethorpe.edu/knottheory/wanted/

1. Reidemeister moves

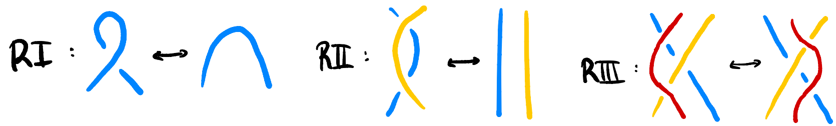
RI : 

Example: 

RII : 

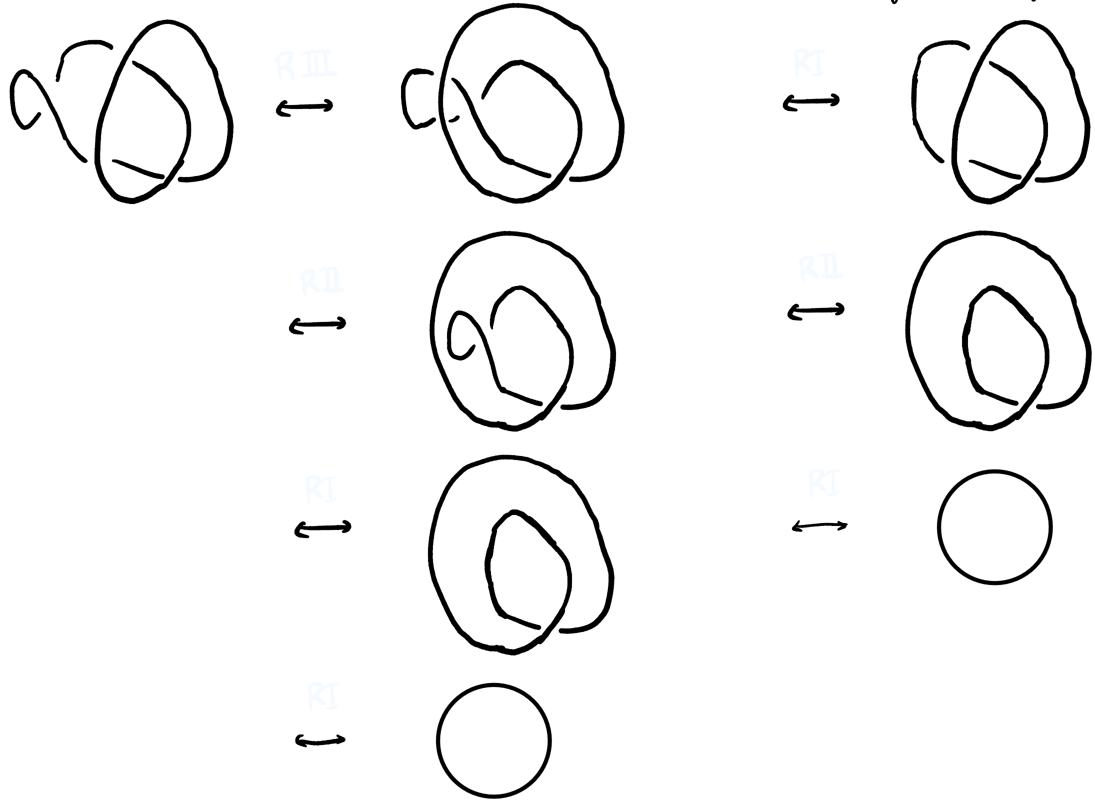
RIII : 

Theorem (Reidemeister): two link diagrams represent the same Knot if and only if they are related by Reidemeister moves.

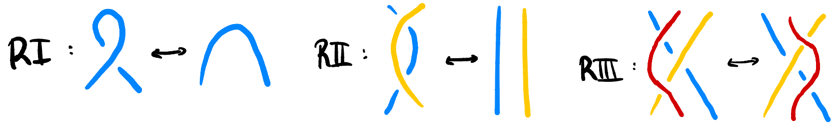


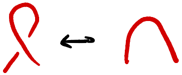
Example :

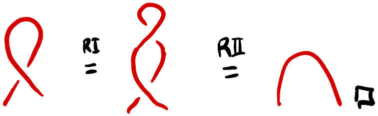
Different way :

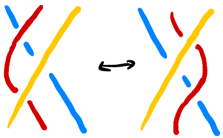


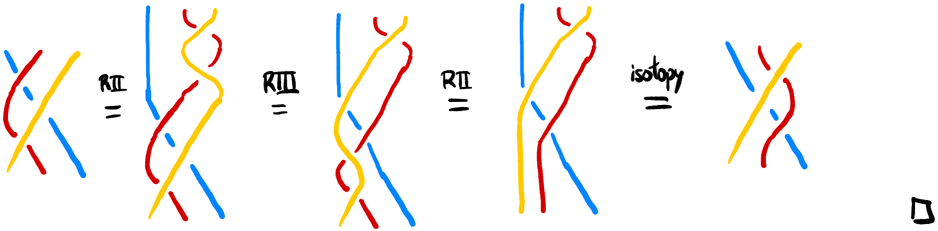
Q? Exercises



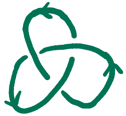
Theorem: RII' :=  follows from RI, RII, RIII.

Proof: 

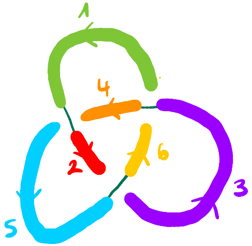
Theorem: RIII' := 

Proof: 

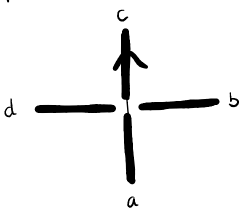
2. Knot theory for computers

Need to translate  ⇝ numbers (Note orientation)

- Step 1: break up the link into edges, and label them 1, ..., n following the orientation:



- Step 2: at each crossing, record the four numbers according to the following rule:



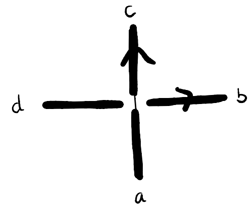
$\longmapsto$ [a, b, c, d]

Example:

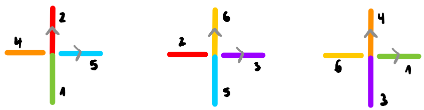
Result: [(1, 5, 2, 4), (5, 3, 6, 2), (3, 1, 4, 6)]

Q?

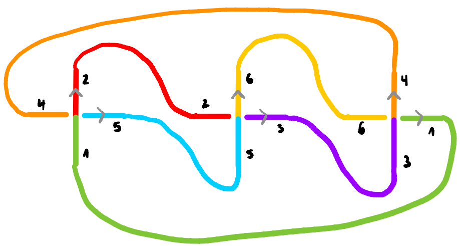
How to get back the link ?

Step 1: Draw a crossing for each group of 4, according to the rule: $[a,b,c,d] \longmapsto$ 

$[(1,5,2,4),(5,3,6,2),(3,1,4,6)]$



Step 2: Match the edges:



Task 0: •

```
1+1
```

• Click Run (or press Shift+Enter):

```
1+1
```

```
2
```

Q?

Task 1:

- Start a new cell
- Import SnapPy:

```
import snappy
```

- Write down the Planar Diagram code:

```
PD= [(1, 5, 2, 4), (5, 3, 6, 2), (3, 1, 4, 6)]
```

- Define a SnapPy link:

```
L_snappy = snappy.Link(PD)
```

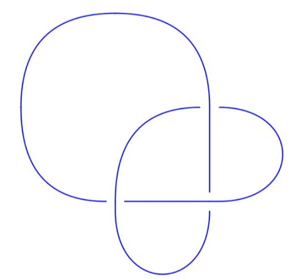
- Make it a Sage link:

```
L=L_snappy.sage_link()
```

- Plot it:

```
L.plot()
```

```
import snappy
PD= [(1, 5, 2, 4), (5, 3, 6, 2), (3, 1, 4, 6)]
L_snappy = snappy.Link(PD)
L=L_snappy.sage_link()
L.plot()
```



Q?

Install SnapPy