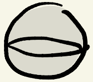
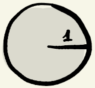
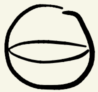
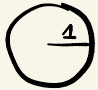
7. Higher dimensions and Lisa Piccirillo's result

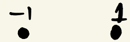
Mathematicians call $B^3 =$  the (solid) 3-ball. It consists of the points (x,y,z) such that $x^2+y^2+z^2 \leq 1$.

Similarly, $B^2 =$ 

in the plane: the points (x,y) such that $x^2+y^2 \leq 1$.

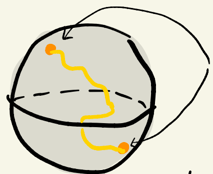
$B^1 =$ 

The "boundary" of each is $S^2 =$  , $S^1 =$ 

, $S^0 =$ 

$x^2+y^2+z^2=1$ $\qquad$ $x^2+y^2=1$ $\qquad$ $x^2=1$

With some imagination, $B^4 : x^2+y^2+z^2+w^2 \leq 1$, $\quad S^3 : x^2+y^2+z^2+w^2=1$.

Note that we can fit some B^1 inside B^3:  on the boundary there is an S^0

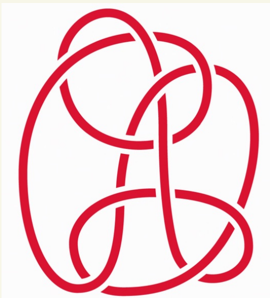
Similarly, we can fit some B^2 inside B^4, and on the boundary there will be an S^1, a <u>knot</u>.

<u>Definition</u>: A knot in S^3 is topologically slice if it can be obtained as the boundary of a B^2 inside B^4. It is smoothly slice (or slice) if the B^2 can be embedded "smoothly".

Out of the thousands of knots with ≤ 12 crossings, mathematicians proved that

$$\text{topologically slice} \longleftrightarrow \text{smoothly slice}$$ (conjecture, 1980s)

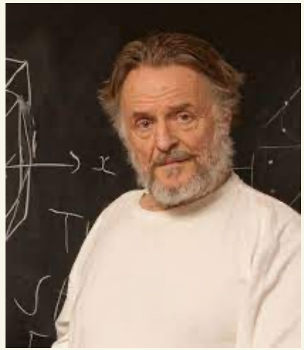
for all but one, Conway's knot:



11_{34}

topologically slice

smoothly slice?



John Conway (1937-2020)

Lisa Piccirillo (2020):

Theorem: The Conway knot is not smoothly slice.

(The proof uses a sophisticated invariant called Rasmussen's s-invariant).

(Identification game)