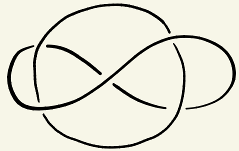
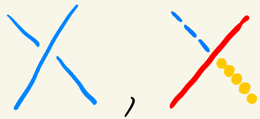
Reminder of Knot theory so far:

- Knots, links and their diagrams



- Knot invariants:
    - Weaker but easier to compute: Tricolorability



-
    - Storger but harder to compute: Jones polynomial

$$\langle \times \rangle = A \langle)(\rangle + A^{-1} \langle \asymp \rangle$$

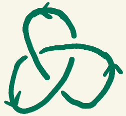
$$\langle \bigcirc \rangle = 1$$

$$\langle L \bigcirc \rangle = (-A^{2} - A^{-2}) \langle L \rangle$$

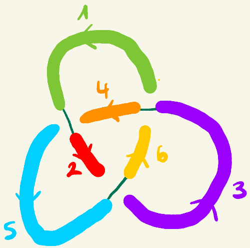
$$J(L) := (-A)^{-3\,\mathrm{writhe}(D)} \cdot \langle D \rangle$$

Today: making computers do the tedious stuff

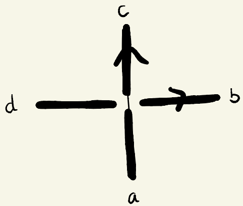
5. Knot theory for computers

Need to translate  $\rightsquigarrow$ numbers

- Step 1: break up the link into edges, and label them $1, \dots, n$ following the orientation:



- Step 2: at each crossing, record the four numbers according to the following rule:



$\mapsto$ $[a, b, c, d]$

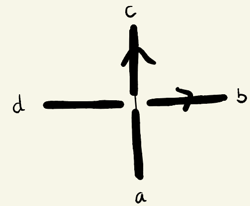
Example:

Result: $[(1, 5, 2, 4), (5, 3, 6, 2), (3, 1, 4, 6)]$

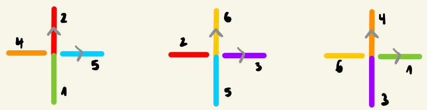
Q?

How to get back the link?

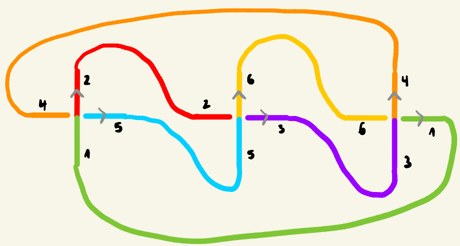
Step 1: Draw a crossing for each group of 4, according to the rule: $[a,b,c,d] \longmapsto$



$[(1,5,2,4),(5,3,6,2),(3,1,4,6)]$



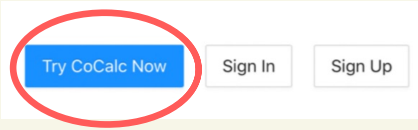
Step 2: Match the edges:

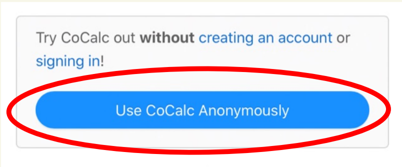


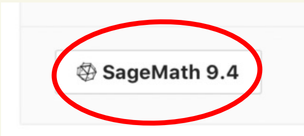
Q?

We will be using SageMath :

No need to download anything, just go to CoCalc : cocalc.com

1. 

2. 

3. 

Q1

Task 0: •
```
1+1
```
• Click Run (or press Shift+Enter):
```
1+1
```
```
2
```
Q?

Task 1:

- Start a new cell
- Import SnapPy:

```
import snappy
```

- Write down the Planar Diagram code:

```
PD= [(1, 5, 2, 4), (5, 3, 6, 2), (3, 1, 4, 6)]
```

- Define a SnapPy link:

```
L_snappy = snappy.Link(PD)
```

- Make it a Sage link:

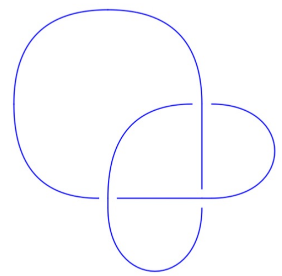
```
L=L_snappy.sage_link()
```

- Plot it:

```
L.plot()
```

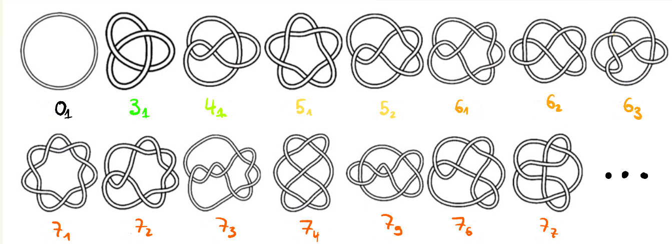
Q?

```
import snappy
PD= [(1, 5, 2, 4), (5, 3, 6, 2), (3, 1, 4, 6)]
L_snappy = snappy.Link(PD)
L=L_snappy.sage_link()
L.plot()
```



Another way to define SnapPy knots:

Recall the table of prime knots:

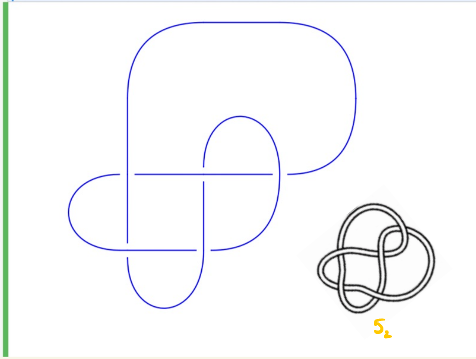



```
L_snappy = snappy.Link('5_2')
```

⇝

```
import snappy

L_snappy = snappy.Link('5_2')
L=L_snappy.sage_link()
L.plot()
```

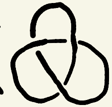


Task 2:

- Start a new cell.
- Choose a knot from the table and plot it.

mirror image of 3_1

Recall that last week we computed: $J\big($  $\big) = -A^{-16} + A^{-12} + A^{-4}$

People usually substitute $A = t^{-\frac{1}{4}}$, so $J\big($  $\big) = -(t^{-\frac{1}{4}})^{-16} + (t^{-\frac{1}{4}})^{-12} + (t^{-\frac{1}{4}})^{-4}$

$$= \boxed{-t^4 + t^3 + t}$$

In Sage, the command for the Jones polynomial is

```
L.jones_polynomial()
```

Task 3:

- Go back to Task 2 and compute the Jones polynomial of L.
  Alternatively, you can define L again by:

```
L_snappy = snappy.Link('3_1')
L=L_snappy.sage_link().mirror_image()
```

Q? Ex