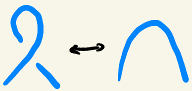
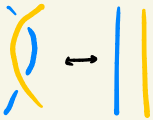
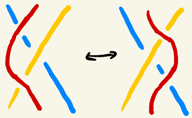
Review of last time

- Knots and links

- Reidemeister moves

RI :  RII :  RIII : 

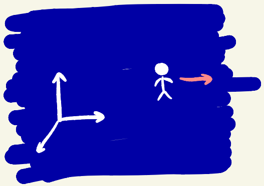
- When two knots are equal

Today: When two knots are different

2. Invariants and tricolorability

Topology, geometry : "Classify" spaces

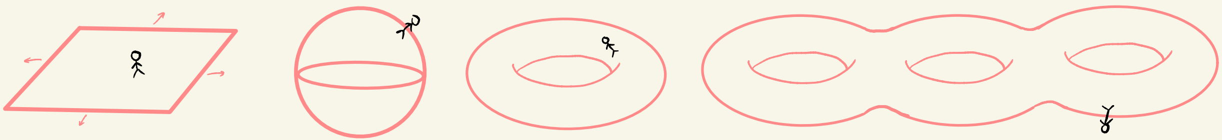
Examples : • 3-manifolds : in how many ways could the universe be shaped?



?

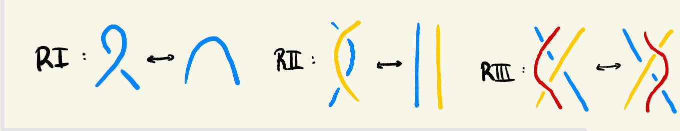
• 2-manifolds : in how many ways could the Earth be shaped?

$\mathbb{R}^2$, S^2, T^2, T_g^2



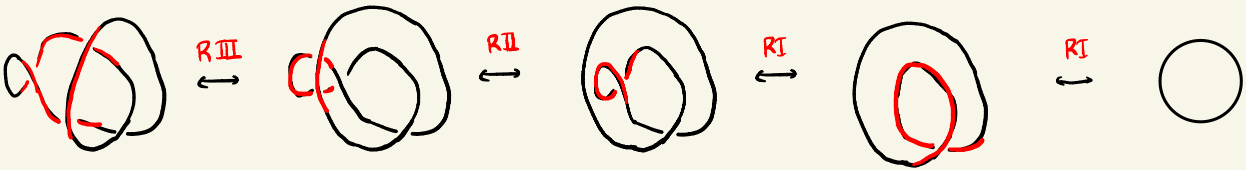
Showing two are equal : elementary transformations

Showing two are different: invariants.

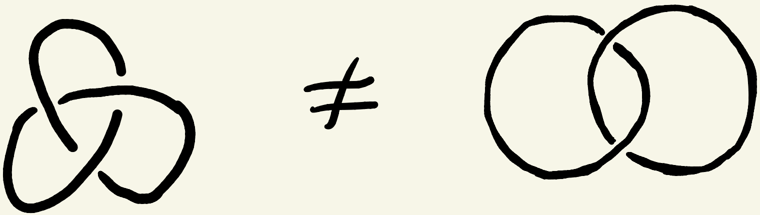


Our example: links

- Showing two are equal:



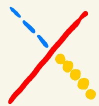
- Showing two are different:



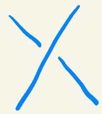
- How about $\neq$ ○ ?

Definition: A link diagram is tricolorable iff one can assign colors to each arc in such a way that:

1) At each crossing either three different colors meet:

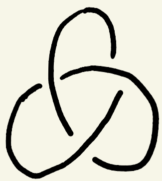


or only one:

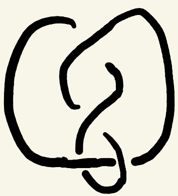


2) At least two colors are used.

Example:



Nonexample:



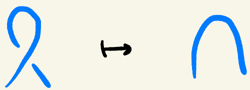
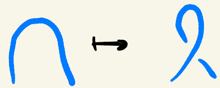
Theorem: tricolorability is a link invariant.

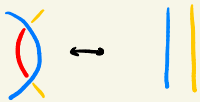
(If L is a link and D_1, D_2 are diagrams for it, then D_1 is tricolorable $\Leftrightarrow$ D_2 is tricolorable)

Proof: A) • Suppose D_1 is tricolorable

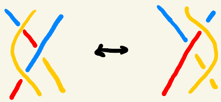
- Reidemeister moves $D_1 \xrightarrow{R?} D_1' \to \dots \to D_2$

- Coloring for D_1':

↳ RI:  RI^{-1}: 

↳ RII: 

(one color: easy)

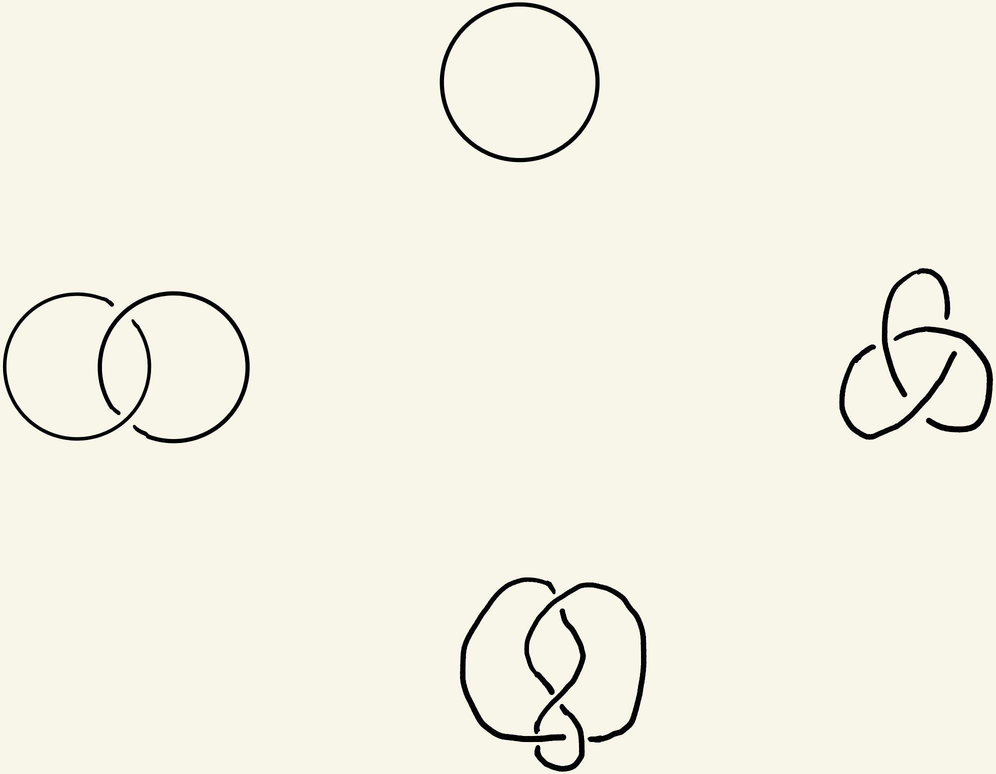
↳ RIII: 

- Repeat for $D_1'', D_1''', \dots, D_2 \Rightarrow D_2$ is tricolorable

B) • Suppose D_1 is <u>not</u> tricolorable. Then if D_2 is tricolorable then D_1 is, a contradiction.

$\Rightarrow D_2$ is not tricolorable.

Can we tell them apart?



Upshot: new ideas are needed

Q? Exercises