

Welcome to Knot Theory

SHP Fall 2021

Instructor: Alvaro Martinez (he/him)

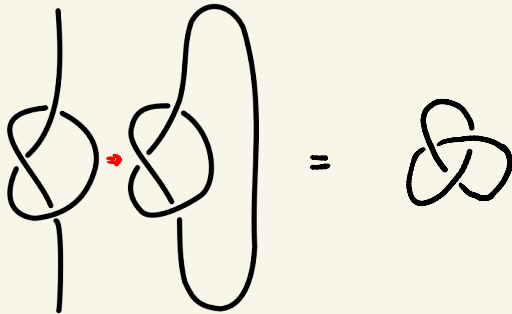
Please fill out the form :

https://docs.google.com/forms/d/e/1FAIpQLSecf6Mgb13PzDThic3GLhdxCwrslyX4T7DIA6SDBBXEOEelQ/viewform?usp=st_link

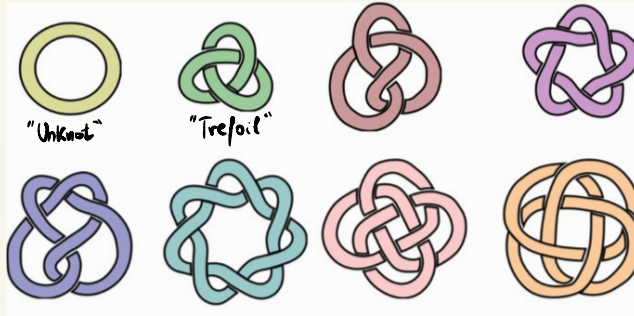
What is a knot?

1. Take a string
2. Knot it however you want.
3. Fuse the ends together.

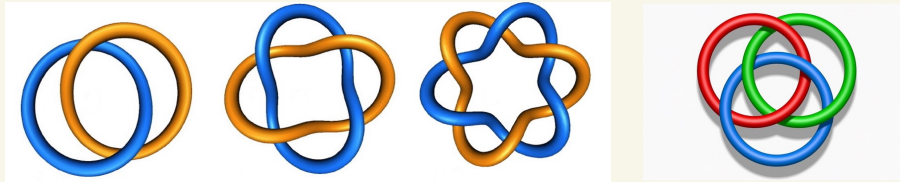
Example:



More examples:



If we knot several strings together, we get a link:

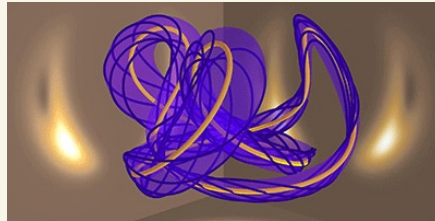


They can get complicated:



Mathematical curiosity?

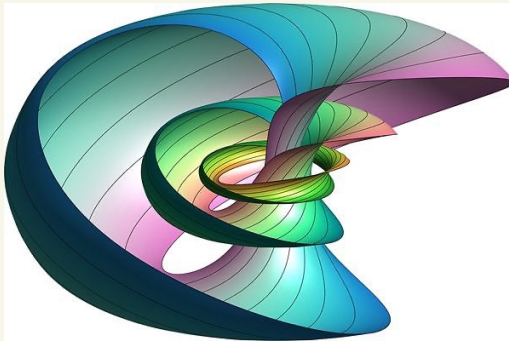
- In electromagnetism:



path in the electromagnetic field

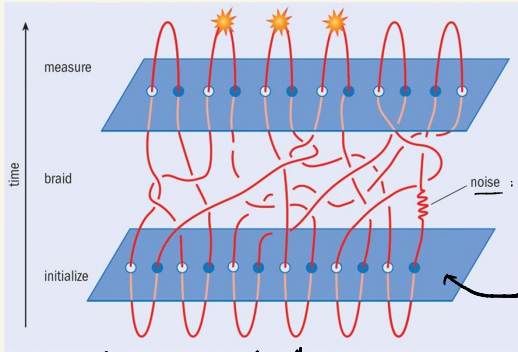
- In mathematics:

all possible "3D-spaces" can be constructed from knots



A 3-manifold in the making.

- In quantum physics

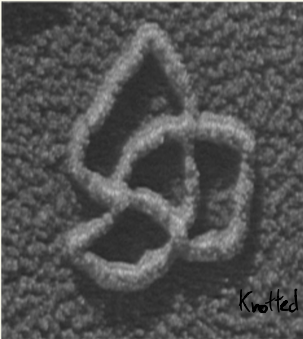


noise : does not affect the computation as long as the knot is the same.

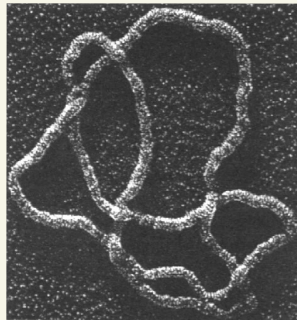
qubits

Fractional quantum Hall effect

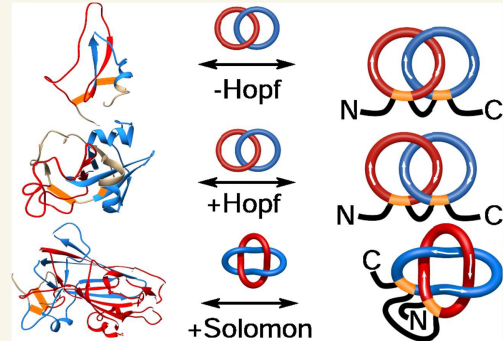
- In biology :



Knotted



Knotted DNA strands

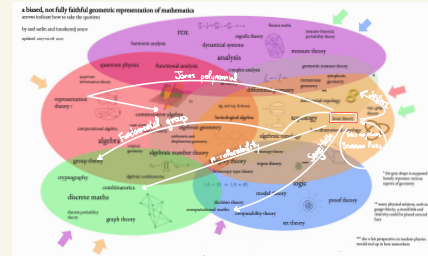


The knottedness of proteins affects their functions.

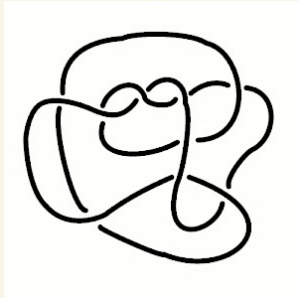
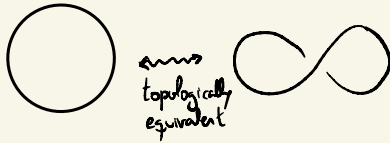
Good news: no need to know any of those things to do knot theory!

Goals:

- Understand knots from a mathematical point of view.
 - ◁ When are two links "the same"?
 - ◁ How can I tell two links apart?
- Explore connections with other fields in math:
(and use knot theory as an excuse to explore them)
- Get a feel for what math research is like.



0. The unknot



https://miro.medium.com/max/600/1*z8xAu-m3jzDqfxjrgohjnw.gif

Poll: which of these are trefoils?

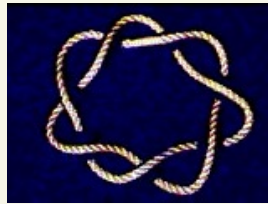
a.



b.



c.



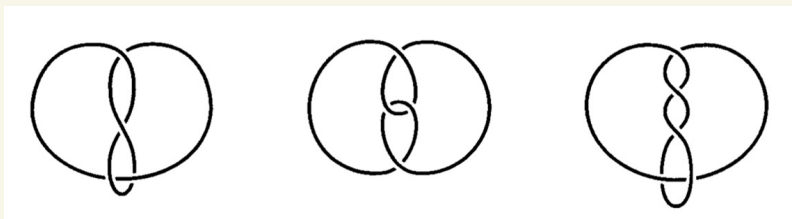
d.



<http://sites.oglethorpe.edu/knottheory/wanted/>

Definition (link diagram): a projection of a link onto the plane so that we can tell under/overcrossings.

Examples:



Theorem: A knot diagram with less than three crossings represents the unknot.

Proof:

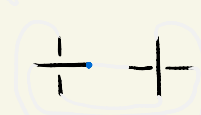
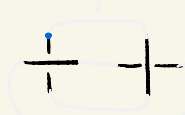
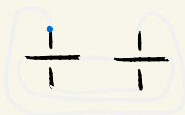
• 0 crossings

• 1 crossing:

• 2 crossings:

- No edge going back:

- Two possibilities:



These are: two unknots

a link

a link

Q?

Poll: what knot is this?

