Lecture 9

- Discussion on disproving statements by counterexample:

  - If $a,b\in\mathbb{Z}$, $a^2>b^2 \Rightarrow a>b$. False: take $a=-2$, $b=1$. Then $\underset{4}{a^2} > \underset{1}{b^2}$ but $\underset{-2}{a} < \underset{1}{b}$.

  - $v_1,\dots,v_k\in\mathbb{R}^m$ l.i./spanning, T inj/surj: See answer key to HW 2.

  - **30.** Find a ~~basis of the subspace of $\mathbb{R}^4$ defined by the equation~~

    Prove that $S=\left\{\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} : \begin{array}{l}2x_1 - x_2 + 2x_3 + 4x_4 = 0. \\ x_1 + x_2 - x_3 - x_4 = 0\end{array}\right\}$ is a subspace of $\mathbb{R}^4$, find a basis for it (You must show it is a basis).

    Proof: $S=\ker(T)$ where $T:\mathbb{R}^4\to\mathbb{R}^2$ has matrix $\begin{pmatrix}2 & -1 & 2 & 4\\ 1 & 1 & -1 & -1\end{pmatrix}$.

    Therefore S is a subspace of $\mathbb{R}^4$. To find a basis, use the method for finding bases of kernels.

  - **37.** Give an example of a 4×5 matrix A with $\dim(\ker A)=3$.

$$\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$$

- Mock midterm solutions (different file).

- More practice questions:

  - **36.** Can you find a 3×3 matrix A such that $\mathrm{im}(A)=\ker(A)$? Explain.

  - Prove or disprove:

    If $T_1:\mathbb{R}^n\to\mathbb{R}^m$ is surjective and $T_2:\mathbb{R}^m\to\mathbb{R}^p$ is surjective then $T_2\circ T_1$ is surjective

    If $T_1:\mathbb{R}^n\to\mathbb{R}^m$ is surjective and $T_2:\mathbb{R}^m\to\mathbb{R}^p$ is injective then $T_2\circ T_1$ is injective.

38. **a.** Consider a linear transformation T from $\mathbb{R}^5$ to $\mathbb{R}^3$. What are the possible values of $\dim(\ker T)$? Explain.

b. Consider a linear transformation T from $\mathbb{R}^4$ to $\mathbb{R}^7$. What are the possible values of $\dim(\mathrm{im}T)$? Explain.

39. We are told that a certain 5×5 matrix A can be written as

$$A = BC,$$

where B is a 5×4 matrix and C is 4×5. Explain how you know that A is not invertible.

31. Let V be the subspace of $\mathbb{R}^4$ defined by the equation

$$x_1 - x_2 + 2x_3 + 4x_4 = 0.$$

Find a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^4$ such that $\ker(T) = \{\vec{0}\}$ and $\operatorname{im}(T) = V$. Describe T by its matrix A.

- Write down the matrix of your favorite linear transformation.
- Find bases for the kernel and image of your answer to the previous question.
- **58.** For which values of the constants b and c is the vector

$$\begin{bmatrix} 3 \\ b \\ c \end{bmatrix} \text{ a linear combination of } \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}, \text{ and } \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}?$$

list?

27. Determine whether the following vectors form a basis of $\mathbb{R}^4$:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \\ -8 \end{bmatrix}.$$

28. For which value(s) of the constant k do the vectors below form a basis of $\mathbb{R}^4$?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ k \end{bmatrix}$$

29. Find a basis of the subspace of $\mathbb{R}^3$ defined by the equation

$$2x_1 + 3x_2 + x_3 = 0.$$