Lecture 5	· · · · · · · · · · · · · ·
Perop: • Characterized linear independence, spanning, bases in terms of ranv	
$v_{2},, v_{n} \in \mathbb{R}^{m}$ are linearly independent \Leftrightarrow rouk $\begin{pmatrix} v_{2},, v_{n} \\ v_{2},, v_{n} \end{pmatrix} = \#ver$	pa (= n)
v₂,,vn ∈ R ^m are a spanning set ⇐> rank (v₂vn) = "dimension	sion of the space $(=m)$
$v_{a}, \ldots, v_{n} \in \mathbb{R}^{m}$ are a basis $\iff m = n = \operatorname{rank} \begin{pmatrix} 1 & 1 \\ v_{a} \cdots v_{n} \\ 1 & 1 \end{pmatrix}$.	
· Defined Linear Traveformations.	
$-A \text{function} T: \mathbb{R}^n \longrightarrow \mathbb{R}^m \text{given by} T\left(\begin{pmatrix} x_n \\ x_n \end{pmatrix}\right) = \begin{pmatrix} Q_{n_1} x_n \\ Q_{n_2} x_n \\ Q_{n_1} x_n \end{pmatrix}$	+ are x2 + + are xn + are x2 + + are xn (+ are x2 + + are xn + are x2 + + are xn Hatrix of the transformation
- Equivalently, a function $T \mathbb{R}^n \longrightarrow \mathbb{R}^m$ given by $T((x_n))$	$\bigg) = \chi_1 Q_1 + \chi_2 Q_2 + \dots + \chi_n Q_n$
	for some fixed vectors $a_{1,,a_n} \in \mathbb{R}^m$
- Equivalently, a function $T:\mathbb{R}^n \longrightarrow \mathbb{R}^m$ such that	T(v+w) = T(v) + T(w) $T(\lambda v) = \lambda T(v)$ $Yv, w \in \mathbb{R}^{n}$, $\lambda \in \mathbb{R}$
Today: More on Pincer transformations and matrix products Examples of finear transformations $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ (good to	(leep in mird).
Example 1: Scaling	· · · · · · · · · · · · · · · ·
Consider the linear transformation $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} 2x_2 \\ 3x_2 \end{pmatrix} = \begin{pmatrix} 2x_2 + 0x_2 \\ 0x_2 + 3x_2 \end{pmatrix}$	Hs associated matrix is $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$
This motivix scales the x-coordinates by 2 and the y-coord	inates by 3
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Example 2 Polation:
Consider
$$T(\frac{x_{1}}{x_{2}}) = (-\frac{x_{2}}{x_{2}}) - (\frac{0x_{2}-x_{1}}{x_{2}+0x_{2}})$$
 It's notice is $(\frac{0}{2}, -\frac{4}{2})$.
This matrix is a conditrollecturize 90° robotion.

Example 3: Sheer:
 $T(\frac{x_{1}}{x_{2}}) = (\frac{x_{2}+x_{1}}{x_{2}}) \longrightarrow (\frac{4}{0}, \frac{4}{0})$

Example 4: Reflection
 $T(\frac{x_{2}}{x_{2}}) = (-\frac{x_{1}}{x_{2}}) \longrightarrow (-\frac{4}{0}, \frac{0}{0})$

Example 4: Reflection
 $T(\frac{x_{1}}{x_{2}}) = (-\frac{x_{1}}{x_{2}}) \longrightarrow (-\frac{4}{0}, \frac{0}{0})$

P. F. D- atime			• •	• •	• •					٠
xample of projection						•	• •			•
$\overline{\top}\left(\begin{pmatrix} X_{\mathbf{a}} \\ X_{\mathbf{b}} \end{pmatrix}\right) = \begin{pmatrix} X_{\mathbf{a}} \\ \mathbf{O} \end{pmatrix}$		• • •						•	•	•
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· · · · · · · · · · · · · ·						•	• •	•	•	•
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) T(6)	••••				•	• •	•	•	•
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· · · · · · · · · · · · · · ·	J 		 h.					•	•	•
Observe: this is the only one that	we've seen	that	loses	inform	ation	•				•
Example 6: $T(x_a) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. This loses (all of the in	bruction	n, .	• •	• •	•	• •	•	•	•
						•		•	•	•
Composing linear transformations				0 0	• •	٠	• •			•
=	ີຄະ	• • •			• •	•		•		•
$Juppoxe I_2 : K \longrightarrow IK and I_2 : IK \longrightarrow$	- K and	e given	by <u>'</u>	• •	• •	•	• •			•
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 & 0 \end{pmatrix}$	1 1	• • •	• •	• •	• •	•	• •	•	•	•
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (rotation) (sheat	1) 1) ar)	· · · ·	· ·	· ·	· ·		· ·		•	•
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ (rotation) \end{pmatrix} \qquad (shee$ and want to compose them one after the of	1) 1) ar) Mer	· · · ·	linear a	· · ·	. T. li		· · ·	•	•	•
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (votation) (sheen and) want to compose them one after the other the other indication: $T_2 \circ T_1$ is linear. Prove		τ <u>,</u> (ν+ω) =	- linear - T ₂ (T		 ↓ . 	16ar T2 0 T	· · · · · · · · · · · · · · · · · · ·	- - - 2°	T2(w).
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (sheen and) want to compose them one offer the o	$f = T_{a} \circ T_{a}$	τ _ε (ν+ω) =	- T ₂ (T ₃	[(v)+7₂((T (.)) -	t, li J ((u)) =	16ar TzoT	L(u) +		T2(w	· · · ·
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (sheen and want to compose them one offer the ot function observation: T ₂ o T ₂ is linear. Proop	1 1 her f: T _a ∘T ₁ T _a ∘T ₁	τ. (ν+ω) = (λν) =	- lineoγ - T ₂ (T ₃ - T ₂ (λ	[(v)+T2([1(v)) =	τ. li J ((u)) =	near T₂⊙T • T₁(v)	L(U) +		Ī2(w)
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (shee and want to compose them one after the of fructal observation: $T_2 \circ T_1$ is linear. Proop Then, what is the matrix of $T_2 \circ T_1$?	1 1 her f T _a ο T ₁ T _a ο T ₁	τ <u>α</u> (ν+ω) = (λν) =	= lineoν = T ₂ (T ₃ = T ₂ (λ	[(v)+72([1(v)) =	τ. l: ↓ ω)) = λΤΣ	16ar T₂=T₂ •T₂(v)	μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ	- - - - - - - - - - - - - - - - - - -	Ī4(w	· · · ·
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$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (shee and want to compose them one after the of Trucial observation: $T_2 \circ T_2$ is linear. Prop Then, what is the matrix of $T_2 \circ T_1$? Idea: only have to look at where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $T_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$ \begin{pmatrix} a \\ 1 \end{pmatrix} \\ AT \end{pmatrix} $ her $ \int \mathbf{T}_{\mathbf{a}} \cdot \mathbf{T}_{4} \\ \int \mathbf{T}_{5} \cdot \mathbf{T}_{4} \\ \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathbf{g} 0 \\ \end{pmatrix} $	τ _α (ν+ω) = (λν) =	$= \frac{1}{T_2} \left(\frac{1}{\lambda} \right)$ $= \frac{1}{T_2} \left(\frac{1}{\lambda} \right)$	[(v)+7₂(([2(v))) =	τ (i J)) =	16ar T₂=T • T⊥(V)			[]_(ω)	· · · · · · · · · · · · · · · · · · ·
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (shee and want to compose them one after the of Trucial observation: $T_2 \circ T_2$ is linear. Prop Then, what is the matrix of $T_2 \circ T_2$? Idea: only have to look at where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and Now $T_2(\begin{pmatrix} 2 \\ 0 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (first column of A)	¹ / ₁) λer β T _a • T ₁ Τ ₃ • T ₄ (°) go	τ _α (ν+ω) = (λν) =	$= \frac{1}{12} \left(\frac{1}{2} \right)$ $= \frac{1}{12} \left(\frac{1}{2} \right)$ $= \frac{1}{12} \left(\frac{1}{2} \right)$	[(v)+7z([1(v)) =	τ. fi J w)) = λ Τ ₂	16ar T₂∘T •T₁(v)	μ μ μ μ μ μ μ μ μ μ μ μ μ μ		T ₄ (w	· · · · · · · · · · · · · · · · · · ·
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (shee and want to compose them one after the of Frucial observation: $T_2 \circ T_2$ is linear. Prop Then, what is the matrix of $T_2 \circ T_1$? Idea: only have to look at where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and Now $T_2(\begin{pmatrix} 2 \\ 0 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (first column of A) and $T_2(\begin{pmatrix} 0 \\ 4 \end{pmatrix}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	¹ / ₁) λ(τ) her f: T ₂ ∘ T ₁ T ₃ ∘ T ₁ (°) go	τ. (ν+ω) = (λν) =	$= \frac{1}{12} \left(\frac{1}{\lambda} \right)^{2}$ $= \frac{1}{12} \left(\frac{1}{\lambda} \right)^{2}$	[(v)+7 ₂ (c [1(v)) =	- λTz	16ar T₂⊙T • T₂(v)	· · · · · · · · · · · · · · · · · · ·	т. 	T ₂ (w	· · · · · · · · · · · · · · · · · · ·
$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (rotation) (shee and want to compose them one after the of Trucial observation: $T_2 \circ T_2$ is linear. Prop Then, what is the matrix of $T_2 \circ T_1$? Idea: only have to look at where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and Now $T_2(\begin{pmatrix} 2 \\ 0 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (first column of A) and $T_2(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	a f T ₁ • T ₁ f T ₁ • T ₁ (°) (°) (°) (°) (°) (°)	τ _α (ν+ω) =	$= \frac{1}{T_2} \left(\frac{1}{\lambda} \right)$ $= \frac{1}{T_2} \left(\frac{1}{\lambda} \right)$ $= \frac{1}{T_2} \left(\frac{1}{\lambda} \right)$	[(v)+T₂((T₂(v))) =	- τ (i - μ - λ Τ.	16ar T₂⊙T • T⊥(V)	· · · · · · · · · · · · · · · · · · ·	$ \frac{1}{1} \frac{1}{2} 1$	T ₂ (w	· · · · · · · · · · · · · · · · · · ·
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nvertibility												•
inition 2: A	finear tra	nformation	τı	has an	inverse	iff there	exists a	nother f	inear t	ransbumat	ion T	ية. ر
such	that T	$T^{-1} = ide$	ntity map	and	T'.T	= identity	map.	• •	• •			•
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The problem is: $T\begin{pmatrix} 0\\ 4 \end{pmatrix} = \begin{pmatrix} 0\\ 0 \end{pmatrix}$, so $T' \circ T\begin{pmatrix} 0\\ 4 \end{pmatrix} = T' \begin{pmatrix} T\begin{pmatrix} 0\\ 4 \end{pmatrix} = T' \begin{pmatrix} 0\\ 4 \end{pmatrix} = T' \begin{pmatrix} 0\\ 4 \end{pmatrix}$.
So such a T'annot exist!
In general, if T "kills" some vector, then T cannot have an inverse
Another problem: the image of each element of \mathbb{R}^2 under T is a multiple of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, so in particular
we cannot have $T \circ T'\binom{\circ}{1} = \binom{\circ}{1}$, since $T \circ T'\binom{\circ}{1} = \overline{T} \left(T'\binom{\circ}{1} \right)$
image of an elevent under T.
The will come back to these ideas taken. In order to play around with interves, it will be useful to
tearn row to compute merrin
tact: Unly square mainces have interfes (we will prove this soon)
How to find the inverse of a matrix (fit exists.)
Suppose we want to find the inverse of A, the matrix of a linear transformation $T \cdot R^{-} - R^{-}$.
Then we seek another square matrix A' such that AA' = Id
In particular, $AA' \left(\frac{4}{9}\right) = \left(\frac{4}{9}\right)^{\frac{1}{9}}$ Gaussian dim Gau figure out the first column of A' 1
Jirst column of A'
Similerly, n linear systems will give all of A', column by column. If no invence exists, one of
the systems will be incompatible.
Example 8: Let us find the inverse of $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Get a system: unknown
$ \begin{pmatrix} 0 & -1 & & 1 \\ 1 & 2 & & 0 \end{pmatrix} \xrightarrow{\mathbf{I} \to \mathbf{I}} \begin{pmatrix} 1 & 2 & & 0 \\ 0 & -1 & & 1 \end{pmatrix} \xrightarrow{\mathbf{I} \to -\mathbf{I}} \begin{pmatrix} 1 & 2 & & 0 \\ 0 & 1 & & -1 \end{pmatrix} \xrightarrow{\mathbf{I} \to \mathbf{I} - 2 \cdot \mathbf{I}} \begin{pmatrix} 1 & 0 & & 2 \\ 0 & 1 & & -1 \end{pmatrix} \implies \begin{pmatrix} 2 \\ -1 \end{pmatrix} $ is the lateralised
$\begin{pmatrix} \mathbf{O} & -\mathbf{I} \\ \mathbf{I} & \mathbf{O} \end{pmatrix} \cdot \mathbf{A}^{\mathbf{I}} \begin{pmatrix} \mathbf{O} \\ \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{O} \\ \mathbf{I} \end{pmatrix}$
$ \begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \xrightarrow{\mathbf{I} \to \mathbf{I}} \begin{pmatrix} 1 & 2 & & 1 \\ 0 & -1 & 0 \end{pmatrix} \xrightarrow{\mathbf{I} \to -\mathbf{I}} \begin{pmatrix} 4 & 2 & & 1 \\ 0 & 4 & 0 \end{pmatrix} \xrightarrow{\mathbf{I} \to -\mathbf{I} - 2 \cdot \mathbf{I}} \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & & 4 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix} 4 & 0 & 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix}$

$\Rightarrow A^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$	et's dreck this: $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$ $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$
$\frac{\text{Romark}}{\text{togetter}} \text{ Notice that solving the togetter as follows:}$	the 2 systems required the same steps. We can put these steps Gaussian etim. ~ 7 (rref(A) (A')
$ \left(\begin{array}{c} 1 & nef (A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ 0 \\ 0 \\ ver again! \\ S $	then the inverse of A is A'. Otherwise, A has no inverse, nated A^{-1} , you never have to do Gaussian elim to solve $Ax = b$ imply, $Ax = b \implies A^{-1}Ax = A^{-1}b \implies x = A^{-1}b$.
1. What is the matrix of the ti What about the projection Are both of them invertible	rangler mation $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ which reflects along the line $y = x$? on onto that same line? (Hint: where do the basis rectors go?) b? Find the inverses of they exist.
2. "Draw" the linear transform Check that AB sends the	mations for $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ and AB as in the facture. e basis vectors to the same images as composing the other two.
3. Find the inverse of the mate	$n \times \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$