Linear Algebra HW Week 1

1. Put the following matrices in RREF:

   (a) $\begin{pmatrix} 2 & 4 & 6 \\ -1 & -2 & 4 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -1 & 5 \end{pmatrix}$

   (d) $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 3 \\ 2 & -1 & -7/2 \end{pmatrix}$

2. For each of the following systems of linear equations, write down the corresponding augmented matrix and find **all** of its solutions (if they exist) by performing Gaussian elimination. In each case, show that your solutions satisfy the system of equations.

   (a) $\begin{aligned} 2x &+& y &&&=& -3 \\ x &+& 2y &&&=& 0 \end{aligned}$

   (b) $\begin{aligned} x &+& y &&&=& 2 \\ -2x &+& -2y &&&=& 0 \end{aligned}$

   (c) $\begin{aligned} 2x &+& 4y &+& 5z &=& 9 \\ 7x &+& 3y &+& -2z &=& 8 \\ 2x &+& 3y &+& 2z &=& 0 \end{aligned}$

   (d) $\begin{aligned} 6x &+& 2y &+& z &=& 1 \\ x &+& 3y &+& z &=& 2 \\ 5x &+& -y & & &=& -1 \end{aligned}$

3. Draw the systems (a) and (b) from the previous exercise on the cartesian plane, making sure to indicate where the solution (or lack thereof) appears in your drawing. Give the geometric interpretation of the systems (c) and (d) in words.

4. Determine all the $2 \times (3+1)$ augmented matrices (2 rows, $3+1$ columns) in RREF, (you may write $*$ for entries that can take any value). Indicate which systems are consistent. Do any of them have a unique solution?