Practive Exam 2

Linear Algebra, Dave Bayer, October 28, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

To be graded, this practice exam must be turned in at the end of class on Thursday, November 4. Such exams will be returned in class on the following Tuesday, November 9. Participation is optional; scores will not be used to determine course grades. If you do participate, you may use your judgement in seeking any assistance of your choosing, or you may take this test under simulated exam conditions.

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let P be the set of all polynomials $f(x)$, and let Q be the subset of P consisting of all polynomials $f(x)$ so $f(0) = f(1) = 0$. Show that Q is a subspace of P.

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[2] Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 1 & -1 & 0 \end{bmatrix}.$$

Compute the row space and column space of A.

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[**3**] The four vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

span a subspace V of $\mathbb{R}^3$, but are not a basis for V. Choose a subset of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ which forms a basis for V. Extend this basis for V to a basis for $\mathbb{R}^3$.

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[4] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which rotates one half turn around the axis given by the vector $(1, 1, 1)$. Find a matrix A representing L with respect to the standard basis

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Choose a new basis $\{\mathbf{v}_1,\ \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$ which makes L easier to describe, and find a matrix B representing L with respect to this new basis.

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[5] Let $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ and $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, and let L be the linear transformation represented by the matrix A with respect to $\{\mathbf{e}_1,\ \mathbf{e}_2\}$, where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad A = \begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}.$$

Find the transition matrix S corresponding to the change of basis from $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$. Find a matrix B representing L with respect to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$.

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