Exam 2

Linear Algebra, Dave Bayer, November 11, 1999

Name: ____________________

ID: ____________________ **School:** ____________________

[**1**] (6 pts)	[**2**] (6 pts)	[**3**] (6 pts)	[**4**] (6 pts)	[**5**] (6 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let P be the set of all degree ≤ 4 polynomials in one variable x with real coefficients. Let Q be the subset of P consisting of all odd polynomials, i.e. all polynomials $f(x)$ so $f(-x) = -f(x)$. Show that Q is a subspace of P. Choose a basis for Q. Extend this basis for Q to a basis for P.

 Continued on page: ______

Problem: ______

Continued on page: ______

[2] Let A be the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}.$$

Compute the row space and column space of A.

Continued on page: ______

Problem: ______

Continued on page: ______

[3] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which reflects through the plane P defined by $x + y + z = 0$. In other words, if $\mathbf{u}$ is a vector lying in the plane P, and $\mathbf{v}$ is a vector perpendicular to the plane P, then $L(\mathbf{u} + \mathbf{v}) = \mathbf{u} - \mathbf{v}$. Choose a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, and find a matrix A representing L with respect to this basis.

 Continued on page: ______

Problem: ______

Continued on page: ______

[4] Let $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ and $\{\mathbf{v}_1,\ \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, and let L be the linear transformation represented by the matrix A with respect to $\{\mathbf{e}_1,\ \mathbf{e}_2\}$, where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 2 \\ -4 & 5 \end{bmatrix}.$$

Find the transition matrix S corresponding to the change of basis from $\{\mathbf{e}_1,\ \mathbf{e}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$. Find a matrix B representing L with respect to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$.

 Continued on page: ______

Problem: ______

Continued on page: ______

[5] Let $\{\mathbf{u}_1,\ \mathbf{u}_2\}$, $\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and $\{\mathbf{w}_1,\ \mathbf{w}_2\}$ be ordered bases for $\mathbb{R}^2$. If

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

is the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1,\ \mathbf{u}_2\}$ to $\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and

$$B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

is the transition matrix corresponding to the change of basis from $\{\mathbf{u}_1,\ \mathbf{u}_2\}$ to $\{\mathbf{w}_1,\ \mathbf{w}_2\}$, express $\mathbf{v}_1$ and $\mathbf{v}_2$ in terms of $\mathbf{w}_1$ and $\mathbf{w}_2$.

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Problem: ______

Continued on page: ______