## Exam 1

$$
\text { Linear Algebra, Dave Bayer, October 7, } 1999
$$

Name:
ID: $\qquad$

## School:

$\qquad$

| $[\mathbf{1}](6 \mathrm{pts})$ | $[\mathbf{2}](6 \mathrm{pts})$ | $[\mathbf{3}](6 \mathrm{pts})$ | $[\mathbf{4}](6 \mathrm{pts})$ | $[\mathbf{5}](6 \mathrm{pts})$ | TOTAL |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.
[1] Solve the following system of equations:

$$
\left[\begin{array}{lll}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
2 \\
0 \\
0
\end{array}\right]
$$

Problem:
[2] Compute matrices giving the number of walks of lengths 1,2 , and 3 between pairs of vertices of the following graph:


Problem:
[3] Express the following matrix as a product of elementary matrices:

$$
\left[\begin{array}{lll}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right]
$$

Problem:
[4] Compute the determinant of the following $4 \times 4$ matrix:

$$
\left[\begin{array}{llll}
1 & 1 & 1 & 0 \\
2 & 2 & 0 & 2 \\
3 & 0 & 3 & 3 \\
0 & 4 & 4 & 4
\end{array}\right]
$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

Problem:
[5] Use Cramer's rule to give a formula for the solution to the following system of equations:

$$
\left[\begin{array}{lll}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
2 a \\
2 b \\
2 c
\end{array}\right]
$$

Problem:

