Final Exam

Linear Algebra, Dave Bayer, December 16, 1999

Name: ______________________________

ID: ____________________ **School:** ____________________

[**1**] (5 pts)	[**2**] (5 pts)	[**3**] (5 pts)	[**4**] (5 pts)	
[**5**] (5 pts)	[**6**] (5 pts)	[**7**] (5 pts)	[**8**] (5 pts)	**TOTAL**

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which projects onto the line $(1, 1, 1)$. In other words, if $\mathbf{u}$ is a vector in $\mathbb{R}^3$, then $L(\mathbf{u})$ is the projection of $\mathbf{u}$ onto the vector $(1, 1, 1)$. Choose a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, and find a matrix A representing L with respect to this basis.

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[2] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

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[3] By least squares, find the equation of the form $y = ax + b$ which best fits the data $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (1, 1)$, $(x_3, y_3) = (3, 1)$.

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[4] Find (s,t) so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

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[5] Find an orthogonal basis for the subspace $w + 2x + 3y + 4z = 0$ of $\mathbb{R}^4$.

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[6] Let A be the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}.$$

Find a basis of eigenvectors and eigenvalues for A. Find the matrix exponential e^A.

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[7] Let A be the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

Find an orthogonal basis in which A is diagonal.

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[8] Find a matrix A so the substitution

$$\begin{bmatrix} x \\ y \end{bmatrix} = A \begin{bmatrix} s \\ t \end{bmatrix}$$

transforms the quadratic form $x^2 + 4xy + y^2$ into the quadratic form $s^2 - t^2$.

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