Exam 1

Linear Algebra, Dave Bayer, October 5, 2000

Please work only one problem per page, starting with the pages provided, and number all continuations clearly. Only work which can be found in this way will be graded.

Please do not use calculators or decimal notation.

[**1**] Solve the following system of equations:

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

[**2**] Express the following matrix as a product of elementary matrices:

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

[**3**] Compute the determinant of the following 4×4 matrix:

$$\begin{bmatrix} 2 & 0 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

What can you say about the determinant of the $n \times n$ matrix with the same pattern?

[**4**] Using Cramer's rule, find x satisfying the following system of equations:

$$\begin{bmatrix} \lambda & 1 & 0 \\ 1 & \lambda & 1 \\ 0 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

[**5**] Give a formula for the matrix which is inverse to:

$$\begin{bmatrix} \lambda & 1 & 0 \\ 1 & \lambda & 1 \\ 0 & 1 & \lambda \end{bmatrix}$$