Reading: Apostol, chapter 10.

Remark on notation: In the plane, Apostol often writes \( \int_C f \, dx + g \, dy \) for the line integral that was expressed in lecture as \( \int_C (f, g) \cdot d\gamma \). It makes sense, as \( \frac{d\gamma}{dt} = \left( \frac{dx}{dt}, \frac{dy}{dt} \right) \), so at least formally, \( d\gamma = (dx, dy) \).

1. Apostol §8.17 (pp. 268–9) *12.
   Hint: compose with a suitable curve and use the chain rule.

2. Apostol §8.22 (pp. 275–7) *3ab (and evaluate explicitly for \( X(s,t) = s+t, Y(s,t) = st \), and \( f(x,y) = e^{x-y} \)), 8, 9, *14, 15.


*5. Let \( U = \{(x,y) \in \mathbb{R}^2 | (x,y) \neq (0,0)\} \) and let \( F : U \to \mathbb{R}^2 \) be given by

\[
F(x,y) = \left( \frac{x+y}{x^2+y^2}, \frac{y-x}{x^2+y^2} \right).
\]

Use the previous problem to show that this is not conservative.

6. Apostol §10.9 (pp. 331–2) 1, 8, *10.

7. Apostol §10.13 (pp. 336–7) 4, *5a, 6, 7.

8. Apostol §10.18 (pp. 345–6) *14.

*9. A radial force field in \( \mathbb{R}^n \) may be expressed as \( F(r) = f(\|r\|)r/\|r\| \). Assuming that \( f \) is a smooth function of one variable, show that \( F \) is conservative.