1. Apostol §8.3 (pp. 245–7) 2. You needn’t do it all, just enough to get the idea.

2. Apostol §8.5 (pp. 251–2) *6, 7, 8.

3. Apostol §8.9 (pp. 255–6) 4, 7, 8, 9, 10, 12, *20, 21, *22.

4. Apostol §8.14 (pp. 262–3) 1, 2, 7, 10, 11.

   Hint for 13: you may use the fact, asserted in class and in Figure 8.8, that for any scalar field \( f : \mathbb{R}^3 \to \mathbb{R} \), its gradient vectors \( \nabla f(x, y, z) \) are perpendicular to the tangent planes of its level surfaces \( f(x, y, z) = c \). We will speak more rigorously about surfaces and tangent planes in the future.

6. Show that if \( U \) and \( V \) are open in \( \mathbb{R}^n \), then so are \( U \cup V \) and \( U \cap V \).

*7. Show that the product \((a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_n, b_n)\) is an open set in \( \mathbb{R}^n \).

*8. Suppose that \( F : \mathbb{R}^n \to \mathbb{R} \) is linear. Show that for any \( x \in \mathbb{R}^n \), the total derivative of \( F \) at \( x \) is just \( F \) itself.

*9. A function \( G : \mathbb{R}^n \to \mathbb{R} \) is called homogeneous* if \( G(tx) = tG(x) \) for all nonzero \( t \in \mathbb{R} \) and all nonzero \( x \in \mathbb{R}^n \). Suppose that \( G \) is homogeneous and continuous.
   (a) Show that \( G(0) = 0 \).
   (b) Show that the directional derivative of \( G \) at \( 0 \) along \( y \) exists for all \( y \in \mathbb{R}^n \).
   (c) Show that \( G \) is differentiable at \( 0 \) if and only if it is linear. Hint: If it’s differentiable, look at a single line through the origin at a time, and show that there it equals its own derivative.

*10. Suppose \( H : \mathbb{R}^n \to \mathbb{R} \) satisfies \(|H(x)| \leq c \|x\|^2\) for some constant \( c \in \mathbb{R} \) and for all \( x \in \mathbb{R}^n \). Show that it is differentiable at \( 0 \) and compute its derivative.

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*Strictly speaking, this is really called homogeneous of degree 1, but never mind.*