**Lemma 2:** If the categories $\alpha(\varnothing)$ and $\alpha(\varnothing')$ are isomorphic, then the concrete categories $(\alpha(\varnothing), s)$ and $(\alpha(\varnothing'), s)$ are concretely isomorphic.

Proof: We first determine a free algebra $P$ of rank 1 (that is basis cardinality 1) in $\alpha(\varnothing)$. This can be done in many ways. For instance $P = \alpha(\varnothing)$ is a free algebra if and only if it is a projective object of the category, and it furthermore has rank 1 if and only if every morphism $P \to P$ is mono.

The functor $\text{mor}(P, -): \alpha(\varnothing) \to \text{Set}$ is a "new underlying set functor" which is naturally equivalent to the standard underlying set functor $s$ on $\alpha(\varnothing)$.

Let $T: \alpha(\varnothing) \to \alpha(\varnothing')$ be an isomorphism. Then $T(P)$ is a free algebra of rank 1 in $\alpha(\varnothing')$, so we also have a "new underlying set functor" $\text{mor}(T(P), -)$ on $\alpha(\varnothing')$ which is naturally equivalent to the standard one.

If one identifies the sets $\text{mor}(P, A)$ and $\text{mor}(T(P), T(A))$ by means of $T$ for each $A \in \alpha(\varnothing)$, then $T$ commutes with these "new underlying set functors", and it is not difficult, using the properties of the standard underlying set functors, to deduce that $\alpha(\varnothing)$ and $\alpha(\varnothing')$ are also concretely isomorphic with respect to the standard underlying set functors.

Lemma 2 states that two categories of the form $\alpha(\varnothing)$ are abstractly isomorphic if and only if they are concretely isomorphic, or in more algebraic terminology: rationally equivalent. This in fact holds for more general primitive classes of algebras; for instance a slight modification of the above proof shows that two Schreier primitive classes (subalgebras of free algebras are free) whose free algebras of rank 1 are not isomorphic to any of higher rank with surjective epis and injective monos are abstractly isomorphic as categories if and only if they are rationally equivalent. Some restriction on the classes considered is however necessary, for it