That is $\text{As supp}(f)$ means that the value of $f$ on any $s F^K$ is already determined by the restriction of $s$ to $A$.

The essential rank of $f$ is defined as $\min\{|A| | A \in \text{As supp}(f)|$. If $\alpha$ is a primitive class (variety) of algebras, define its rank to be the supremum of the essential ranks of all $\alpha$-algebraic operations. Since the essential rank of an algebraic operation is always less than the dimension ($\Sigma^\alpha$ [3]) of $\alpha$, this supremum exists.

Now let $f$ be any element of the free algebra $F(X, \alpha)$ with basis $X$ of the primitive class $\alpha$, and let $A$ be any algebra in $\alpha$. One can define an $X$-ary operation $f^A$ on $A$ by $f^A(s) = \varphi(f)$ for any $s A^X$. Here $\varphi$ is the homomorphic extension of $\alpha$ to a homomorphism of $F(X, \alpha)$ to $A$. $f^A$ is an algebraic operation on $A$; in fact $f \mapsto f^A$ is a surjective homomorphism of $F(X, \alpha)$ onto the algebra $H^K(A)$ of all $X$-ary algebraic operations on $A$ (see for instance [1]).


THEOREM: If the categories $\alpha(\Delta)$ and $\alpha(\Delta')$ are isomorphic, then $\Delta$ and $\Delta'$ are equivalent types.

We shall prove this theorem in two steps. Let $s$ denote the canonical underlying set functors on $\alpha(\Delta)$ and $\alpha(\Delta')$.

LEMMA 1: If the concrete categories $\alpha(\Delta), s)$ and $\alpha(\Delta'), s)$ are concretely isomorphic (that is isomorphic by a functor which preserves underlying sets), then $\Delta$ is equivalent to $\Delta'$.

Proof: Let $\Delta = (X, \Delta)'$. It suffices to show that the knowledge of the category $\alpha(\Delta)$ and its underlying set functor is sufficient to recover $\Delta$ up to equivalence. Now the definition of free algebra