1. Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ are both $C^1$. Suppose $g \circ f$ and $f \circ g$ are both the identity linear transformation on $\mathbb{R}^2$ (but $f$ and $g$ are not linear). For any $a \in \mathbb{R}^2$, show that both Jacobian matrices $Df(a)$ and $Dg(f(a))$ are invertible. Find the relation between $Df(a)$ and $Dg(f(a))$.

2. Apply the mean value theorem we discussed in class today to prove the following. Suppose $f$ is differentiable on the open interval $(-\epsilon, 1 + \epsilon)$ for some $\epsilon > 0$. Show that there exists $c \in (0, 1)$ such that

$$f(1) = f(0) + f'(0) + \frac{f''(c)}{2}.$$ 

Hint: consider the function

$$h(t) = (1 - t)^2(f(1) - f(0) - f'(0)) + f(t) + f'(t)(1 - t).$$