Honors Math B review information and practice problems for the final exam

May 4, 2014

Instructions: The final exam covers all the material taught in this semester up to \S 12.21. More emphasis will be placed on Chapter 10 (excluding \S 10.19, 10.20, 10.21), Chapter 11 (excluding \S 11.16, 11.17, 11.18, 11.23, 11.24, 11.25), and Chapter 12 (excluding \S 12.16, 12.17, 12.18). The actual test will not involve complicated integral evaluations, but you need to know how to set up an integral that include all the components.

There will be 5 true-false questions and 5 long problems in the final exam. The following problems are provided for extra practice.

1. Let $S$ be the surface in $\mathbb{R}^3$ given parametrically by $x(u,v) = u + v + v^2$, $y(u,v) = u + v$, $z(u,v) = v$, where $0 \leq v \leq 1$ and $0 \leq u \leq v$.
   
   (a) Find a unit normal $\mathbf{n}$ of $S$ as a vector function of $u$ and $v$.

   (b) Find the area of $S$.

2. A solid $T$ is described as the region in $\mathbb{R}^3$ inside the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$. The density is $\delta(x, y, z) = 2z$.
   
   (a) Express the mass of $T$ as iterated integrals in both SPHERICAL and CYLINDRICAL coordinates.

   (b) Evaluate whichever integral you prefer.

3. Convert the following integral into an integral in polar coordinate system and evaluate it.

   $$\int_0^2 \int_{\sqrt{8-x^2}}^{\sqrt{8-x^2}} (x^2 + y^2)^{\frac{3}{2}} dy dx = \int_0^2 \int_{\frac{\pi}{2}}^{\pi} (\cdot) dr d\theta$$

4. Let $L$ represent the line segment joining points $(0, 0, a)$ and $(0, b, 0)$ for $a > 0$ and $b > 0$. Rotate $L$ about the $z$ axis to obtain a surface. Sketch the surface. Set up a double integral to compute the area of the surface and evaluate it.

5. Every curve in this problem is oriented in the counterclockwise direction. Apply Green’s Theorem in the following calculations.
(a) Let \( C_1 \) be the square with vertices \((1,1), (-1,-1), (-1,1)\) and \((1,-1)\) and \( f_1 = (x + y^2, y + x^2) \), calculate \( \int_{C_1} f_1 \cdot \mathbf{T} \, ds \).

(b) Let \( C_2 \) be the circle \( x^2 + y^2 = 9 \) and \( f_2 = (y^2 - x^2, x^2 + y^2) \), calculate the flux \( \int_{C_2} f_2 \cdot \mathbf{n} \, ds \).

6. Let \( B \) be the region bounded by \( x + 2y = \pi, x + 2y = 0, x - 2y = 0, \) and \( x - 2y = \pi \). Use change of variable formula to rewrite the following integral in \((u, v)\) variables so that all the integration limits are constant numbers. Evaluate the integral.

\[
\int \int_B (x - 2y)^2 \sin(x + 2y) \, dA = \int_? \int_? (?) \, dudv
\]

7. Let \( f \) be the vector field \( f = (z, 2x, y) \). Let \( C \) be the ellipse in which the plane \( z = y + 3 \) intersects the cylinder \( x^2 + y^2 = 1 \). Orient the ellipse counter-clockwise when viewed from above.

(a) Calculate the integral \( \int_C f \cdot d\alpha \) using Stoke’s theorem.

(b) DIRECTLY compute the same line integral \( \int_C f \cdot d\alpha \) by parametrizing \( C \) first.

8. Consider the circular cone of height \( h \), radius \( R \), with vertex at \((0,0,h)\), given, in cylindrical coordinates by \( z = h - \frac{h}{R} r \), for \( z \geq 0 \). The surface does NOT include the base disc in the \( xy \)-plane. Suppose that the vector field \( F = \text{curl} \, G \) for some other vector field \( G \), where \( F = (P(x,y,z), Q(x,y,z), 27) \). Compute the flux of \( F \) through the lateral surface of the cone, with respect to the upward pointing normal. You do not need to know what \( G, P \) or \( Q \) are to do this problem; find a simpler surface.