Algebraic topology

Discussion 4 Section 4.1 of Hatcher (not including the last subsection “CW approximation”).

Homework 4. Due Monday, October 9.

1. Show that \( \pi_n(X, A) \) is abelian if \( n \geq 3 \).

2. Prove directly that \( \pi_n(X, A) \cong \pi_{n-1}(A) \) if \( X \) is contractible.

3. Show that the image of the homomorphism

\[
\pi_2(X, x_0) \to \pi_2(X, A; x_0)
\]

in the homotopy long exact sequence of the pair \( (X, A) \) lies in the center of \( \pi_2(X, A; x_0) \).

4. If \( A \) is a retract of \( X \), the maps in the homotopy long exact sequence have the following properties:

- \( \pi_n(A) \to \pi_n(X) \) is a monomorphism,
- \( \pi_n(X) \to \pi_n(X, A) \) is surjective,
- \( \pi_n(X, A) \to \pi_{n-1}(A) \) is the zero map.