Sample Midterm 2
Calculus 1
Exam: November 11, 2004

Comments: same as for Midterm 1. In particular, always write solution, not just an answer, and cross out anything the grader should ignore. Also, note that this “Sample Midterm” is not comprehensive of everything we have covered, but rather like the midterm covers selections from the relevant topics. Also, I didn’t include any more practice problems on L’Hospital’s Rule or on Optimization, since I thought that the last homework included plenty of questions for practicing those techniques.

(1) Let \( f(x) = x^3 - 3x^2 - 9x + 7 \).
   (a) Calculate \( f' \)
   (b) Calculate \( f'' \)
   (c) On which intervals does \( f \) increase? decrease?
   (d) On which intervals is \( f \) concave up?

(2) The following is a graph of a function \( g(x) \).

   Give the interval(s) on which:
   (a) \( g(x) \) is increasing
   (b) \( g(x) \) is decreasing
   (c) \( g'(x) \) is increasing
   (d) \( g'(x) \) is decreasing

(3) Let \( f(x) = \frac{x + 4}{x^2 - 12} \)
   (a) Find the asymptotes of \( f \)
   (b) Compute \( f' \) and find the critical points of \( f \)
   (c) Determine where \( f \) is increasing and where it is decreasing.
   (d) Which of the critical points you found in (b) are local maximums? local minimums? neither?
   (e) Using the information of parts (a)–(d), sketch the graph of \( f \).

(4) For \( f(x) = 2\sqrt{x} + x^3 - 2 \) compute the following:
   (a) \( f'(x) \)
   (b) Find the equation for the tangent line to the graph of \( f(x) \) at the point \((1,1)\).
   (c) Use the linear approximation for \( f(x) \) near \( x = 1 \) to approximate \( f(1.1) \).

(5) A car moving at 60 mi/h along a straight road passes under a weather balloon rising vertically at 15 mi/h. If the balloon is 1 mi up when the car is directly beneath it, how fast is the distance between the car and the balloon increasing 1 minute later?