CALCULUS 3 SPRING 2005: SAMPLE MIDTERM 1

EACH QUESTION IS WORTH 10 POINTS. DO ALL QUESTIONS. NO CALCULATORS

(1) Set up an integral for the area that is inside the circle \( r = 2 \cos \theta \) and outside the circle \( r = 1 \). Do not evaluate the integral.

(2) Find the cosine of the angle between a diagonal of a cube (i.e., a line joining opposite vertices through the center of the cube) and one of its edges.

(3) For \( P = (1, 2, 1), Q = (-1, 3, 2), R = (1, 1, 1) \) compute:
   (a) The area of the triangle \( PQR \),
   (b) Two unit vectors perpendicular to the plane through the points \( P, Q, R \),
   (c) An equation of the plane passing through the points \( P, Q, R \).

(4) Find an equation of a sphere if one of its diameters has endpoints \((3, -1, 4)\) and \((-1, -3, 0)\).

(5) Convert the equation \( \rho = 2 \sin \phi \cos \theta \) from spherical to cartesian coordinates and identify the surface it describes.

(6) Identify and sketch the graphs of the surfaces
   (a) \( z = -\sqrt{1 - x^2} \).
   (b) \( x^2 + 4y^2 + z^2 = 2x - 4z - 1 \).