This test is all T/F and multiple guess. The last page is an answer sheet. For each problem, circle the correct letter on the answer sheet. You should turn in only the answer sheet. Some questions are harder than others, so you may want to do the easy questions first. If you don’t know a given answer, you should guess. The test is a total of 70 points. Good luck!

1. (2 pts each) True/False. Mark the correct box, T or F, on your answer sheet.

(a) A plane is topologically the same as a sphere.
(b) \(K_{10}\) has 45 edges.
(c) You must use at least 4 colors to color any nonplanar graph.
(d) There is an Euler circuit on \(K_{4,10}\).
(e) \(K_4\) is planar.
(f) Consider graphs with no multiple edges or loops. The number of graphs with 5 vertices and 3 edges is equal to the number of graphs with 5 vertices and 7 edges.
(g) Every loop on a torus is contractible.
(h) If two different planar diagrams represent the same underlying graph, then the dual diagram of each must be the same.
(i) For any five vertices of a planar graph, some two of the five do not share an edge.

Consider the following situation for the next 4 T/F questions.

Your apartment building has 6 floors, numbered 1—6. The elevator in your apartment building is broken. If it stops on an even-numbered floor, then its next stop must be to an odd-numbered floor. Similarly, if it stops on an odd-numbered floor, then its next stop must be an even-numbered floor. (So, for example, if you are on the 4th floor, you can directly take the elevator only to floors 1, 3 or 5; and if you are on the 3rd floor, the elevator’s next stop must be 2, 4 or 6.) Model this situation by a graph: Consider each floor as a vertex, and draw an edge between two vertices only when you can directly take the elevator from one floor to the other.

(j) There is an Euler circuit on this graph.
(k) There is a Hamiltonian circuit on this graph.
(l) This graph is planar.
(m) This graph has a circuit which passes through exactly 5 edges (in other words a path which starts at a vertex, goes through 5 edges to end at the original vertex).
2. (3 pts each) Multiple Guess. Circle the correct letter, A—E, on your answer sheet.

(a) “Euler” rhymes best with
   A. Ferris Bueller
   B. water cooler
   C. Vlad the Impaler
   D. double boiler
   E. Freeto-Lay

(b) A connected planar graph diagram has 15 edges and 8 vertices. It breaks the plane up into how many regions?
   A. 9
   B. 10
   C. 11
   D. 12
   E. We can’t tell exactly. It depends on more specific information about the diagram.

(c) A connected planar graph diagram has 8 edges and 4 vertices. How many vertices does its dual graph have?
   A. 4
   B. 5
   C. 6
   D. 7
   E. 8