1. (10 points) Solve the differential equation

\[ y' + \frac{3y}{x} = 4. \]

2. (10 points) Let \( x > 0 \). Solve the differential equation

\[ y' = \sqrt{xy}; \quad y(1) = 1. \]

3. (10 points) At what interest rate (expressed as a decimal) should money be invested if it is to double after 10 years, compounded continuously:
   (a) \( \log_2(10) \)  
   (b) \( \ln(2)/10 \)  
   (c) \( \sqrt[10]{2} - 1 \)  
   (d) \( \ln(10)/2 \)  
   (e) \( \ln(\sqrt[10]{2} + 1) \).

4. (10 points) Find the three complex numbers which are cube roots of \(-1\).

5. (10 points) Use the Cauchy-Riemann equations to show that \( f(z) = |z|^2 \) is not holomorphic.

6. (10 points) Let \( C \) be the unit circle. Which of the following is equal to

\[ \int_C \sin^2(z) \, dz \]

   (a) \( 2\pi i \)  
   (b) \( \pi^2 \)  
   (c) \( \pi^2/3 \)  
   (d) \( \sin^3(2\pi i)/3 \)  
   (e) \( 0 \).

7. (10 points) Let \( C \) be the circle of radius \( 2\pi \), centered at the origin. Evaluate

\[ \int_C \frac{\sin(z)}{z - (\pi/3)} \, dz \]

8. (10 points) In the region in the plane where it is defined (i.e., \( \cos y \neq 0 \)) prove or disprove that \( F(x, y) = (\tan y, x \sec^2 y) \) is conservative.
9. **(10 points)** Rewrite this integral by changing the order of integration to \(dz\; dx\; dy\):

\[
\int_0^1 \int_0^x \int_0^{x^2+y^2} f(x, y, z) \; dz \; dy \; dx.
\]

10. **(10 points)** Suppose that \(F\) is a vector field in 3-space everywhere perpendicular to a surface \(S\) with boundary \(C\). Show that

\[
\int \int_S (\nabla \times F) \cdot dS = 0
\]

11. **(10 points)** Suppose that \(\text{div}(F) > 0\) inside the unit ball, \(x^2 + y^2 + z^2 \leq 1\). Show that \(F\) cannot be everywhere tangent to the surface of the sphere.

LONGER QUESTIONS

12. **(15 points)** Let \(R\) be the square with vertices \((0, 2), (1, 1), (2, 2), (1, 3)\). Use the change of variables \(u = x - y\) and \(v = x + y\) to evaluate

\[
\int \int_R (x - y)/(x + y) \; dA
\]

13. **(20 points)** An open bottle \(B\) lies on the \(xy\)-plane (it fell since the last midterm). Its volume is 750 ml. Its lip (or boundary) is the circle \(\{x^2 + (z - 1)^2; \; y = 10\}\). Let \(F(x, y, z) = (x + y^2, y, x^2 + 1)\). Compute

\[
\int \int_B F \cdot dS
\]

14. **(20 points)** Let \(\rho(x, y, z) = z^2\) be the density of the cylinder \(x^2 + y^2 \leq 1\) inside the sphere \(x^2 + y^2 + z^2 \leq 4\) (including the curved caps of the cylinder). Compute the total mass.

15. **(15 points)** Let \(S\) be the surface parameterized by \(r(u, v) = (u \cos v, u \sin v, u); \; 0 \leq u \leq 1; \; 0 \leq v \leq 2\pi\). Compute

\[
\int \int_S z(x^2 + y^2) \; dS
\]

16. **(20 points)** (a.) Compute the following surface integral directly and (b.) verify the answer by applying one of our theorems. \(E\) is the solid cylinder \(x^2 + y^2 \leq 1; \; 0 \leq z \leq 1\). Let \(F(x, y, z) = (x, y, -z)\). (Note: \(\partial E\) consists of the cylindrical side as well as the flat top and bottom.)

\[
\int \int_{\partial E} F \cdot dS
\]

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