Note on “Parity and symmetry in intersection and ordinary cohomology” [SZ16]

As mentioned before [SZ16, Proposition 3.3.12], the results of [SZ16, Section 3.2] have analogues over an arbitrary field k in which ℓ is invertible.

Definition. Let X be a Deligne-Mumford stack of finite presentation over k. We say that $A \in \mathrm{D}_c^b(X, \overline{\mathbb{Q}}_\ell)$ is *split admissible* if it is split [SZ16, Definition 3.2.1] and if the pullback of A to $X \otimes_k \bar{k}$ is admissible semisimple [SZ16, Definition 3.3.1].

Let $\sigma = \pm 1$, $w \in \mathbb{Z}$. We denote by $\mathrm{D}_\sigma^w(X, \overline{\mathbb{Q}}_\ell) \subseteq \mathrm{Ob}(\mathrm{D}_c^b(X, \overline{\mathbb{Q}}_\ell))$ the subset consisting of split admissible complexes A such that ${}^p\mathcal{H}^i A$ is $(-1)^{w+i}\sigma$-self-dual with respect to $K_X(-w-i)$ for all i. We denote by $\mathrm{D}_\mathrm{d}^w(X, \overline{\mathbb{Q}}_\ell) \subseteq \mathrm{Ob}\Big(\mathrm{D}_c^b(X, \overline{\mathbb{Q}}_\ell) \times \mathrm{D}_c^b(X, \overline{\mathbb{Q}}_\ell)\Big)$ the subset consisting of pairs (A, B) of split admissible complexes such that ${}^p\mathcal{H}^i A$ is isomorphic to $(D_X\, {}^p\mathcal{H}^i B)(-w-i)$ for all i.

The following is a generalization of [SZ16, Theorems 3.2.3, 3.3.7]. The proof is the same as in [SZ16], with the relative hard Lefschetz theorem provided by [SZ16, Proposition 3.3.10].

Theorem. *Let $f\colon X \to Y$ be a proper morphism of Deligne-Mumford stacks of finite presentation over k, where Y has finite inertia. Then $\mathrm{R}f_*$ preserves D_σ^w and D_d^w.*

In particular, $\mathrm{R}f_*$ preserves split admissible complexes.

Corrections to [SZ16]**.** Definition A.2.1: Read “equal” for “isomorphic”.

In the proof of Proposition 4.3.1, instead of [Del80, 1.7.9] which depends on global equations, one should use the more canonical construction of [Del80, 1.7.10]. The assumption on the existence of global equations is thus superfluous. In the definition of U_n, read “$\#K > n$” for “$\#K \geq n$”. See [SZ18, Proposition 4.3.1] for details.

Reference

[Del80] P. Deligne, *La conjecture de Weil. II*, Inst. Hautes Études Sci. Publ. Math. **52** (1980), 137–252 (French). MR601520 (83c:14017)

[SZ16] S. Sun and W. Zheng, *Parity and symmetry in intersection and ordinary cohomology*, Algebra Number Theory **10** (2016), no. 2, 235–307, DOI 10.2140/ant.2016.10.235.

[SZ18] ———, *Parity and symmetry in intersection and ordinary cohomology* (2018). arXiv:1402.1292v7.