

CALCULUS III — Midterm Sample

Instructions: You have 90 minutes to complete all of the six problems. The exam is worth 100 points total. Calculators and textbooks are prohibited. Simplify your answer as much as you can, but no decimal answers please.

1. (10 pts.) When does the equation

$$x^2 + y^2 + z^2 + x - 4y - z - a + 5 = 0$$

represent a sphere? What is the radius when it does?

2. (20 pts.) Let $P(1, 1, 1)$, $Q(2, 2, 2)$, $R(-1, -2, -3)$ be three points in the space. Find the angle $\angle RPQ$ and the area of the triangle RPQ .
3. (20 pts.) Find a parametric equation of the line passing through the point $(1, 1, 1)$ and parallel to the following two planes:

$$H_1 : 2x - y - z + 1 = 0, \quad H_2 : x - y + z + 10 = 0.$$

And find the distance from the line to the plane H_1 .

4. (10 pts.) What is the surface defined by the spherical equation $\rho = 2 \cos \phi$?
5. (20 pts.) Consider the curve represented by the vector function

$$\mathbf{r}(t) = \langle e^t, \sqrt{2}t, e^{-t} \rangle$$

and two point $P(1, 0, 1)$ and $Q(e, \sqrt{2}, e^{-1})$ on the curve. Complete the following questions:

- (1) Find a parametric equation for the tangent line to the curve at P .
 - (2) Find the arc length between P and Q .
 - (3) Find the curvature at P .
 - (4) Find an equation of the normal plane at P .
6. (20 pts.) Let $ABCD$ be a parallelogram. Prove that

$$|AC|^2 + |BD|^2 = |AB|^2 + |BC|^2 + |CD|^2 + |DA|^2.$$